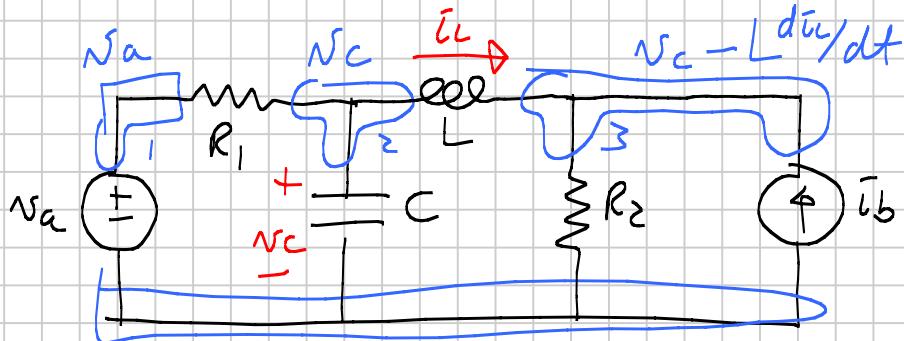


ECE 141 SPRING 2009 TEST 2

Note Title

I)



Typically use capacitors voltages + inductors currents

$$KCL_{n2}: \frac{v_c - v_a}{R_1} + C \frac{dv_c}{dt} + i_L = 0$$

$$\frac{dv_c}{dt} = -\frac{1}{R_1 C} v_c - \frac{1}{C} i_L + \frac{1}{R_1 C} v_a$$

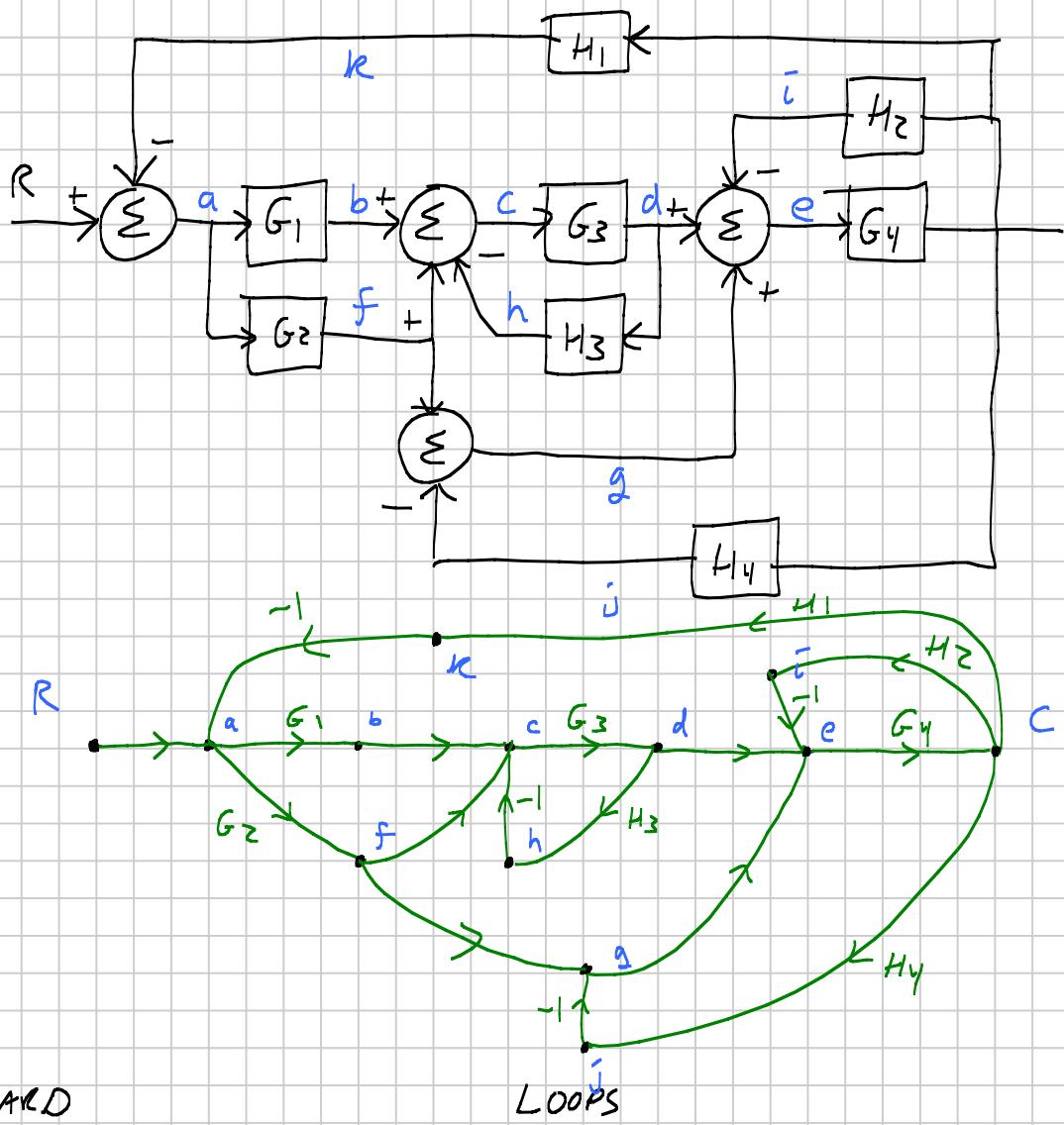
$$KCL_{n3}: -i_L + \frac{(v_c - L \frac{di_L}{dt})}{R_2} - i_b = 0$$

$$-R_2 i_L + v_c - L \frac{di_L}{dt} - R_2 i_b = 0$$

$$\frac{di_L}{dt} = \frac{1}{L} v_c - \frac{R_2}{L} i_L - \frac{R_2}{L} i_b$$

$$Z = \begin{bmatrix} v_c \\ i_L \end{bmatrix} \begin{bmatrix} \bullet \\ v_c \\ i_c \end{bmatrix} = \begin{bmatrix} -\frac{1}{R_1 C} & -\frac{1}{C} \\ \frac{1}{L} & -\frac{R_2}{L} \end{bmatrix} \begin{bmatrix} v_c \\ i_L \end{bmatrix} + \underbrace{\begin{bmatrix} \frac{1}{R_1 C} & 0 \\ 0 & -\frac{R_2}{L} \end{bmatrix} \begin{bmatrix} v_a \\ i_b \end{bmatrix}}_{}$$

II)



FORWARD

$$\begin{array}{ll} T_1 & RabcdeC \\ T_2 & RafcddeC \\ T_3 & Ra^fge^eC \end{array}$$

$$\begin{array}{l} G_1 G_3 G_4 \\ G_2 G_3 G_4 \\ G_2 G_4 \end{array}$$

$$\begin{array}{lll} L_1 & abcdeCja & -G_1 G_3 G_4 H_1 \\ L_2 & afcdedCja & -G_2 G_3 G_4 H_1 \\ L_3 & a^fge^eCa & -G_2 G_4 H_1 \\ L_4 & cdhc & -G_3 H_3 \\ L_5 & eCie & -G_4 H_2 \\ L_6 & eCjge & -G_4 H_4 \end{array}$$

$$\Delta = | -L_1 - L_2 - L_3 - L_4 - L_5 - L_6 + L_3 L_4 + L_4 L_5 + L_5 L_6 |$$

$$\Delta_1 = 1$$

$$\Delta_2 = 1$$

$$\Delta_3 = 1 - L_4$$

$$T = \frac{T_1 \Delta_1 + T_2 \Delta_2 + T_3 \Delta_3}{\Delta}$$

III)

$$(1) T = \frac{s^2 + 2}{s^4 + 2s^3 + 8s^2 + 4s + 1}$$

3L, 2R

$\epsilon > 0$

\downarrow

L

\downarrow

L

\downarrow

R

\downarrow

R

\downarrow

L

$$2) T = \frac{G}{1+G} = \frac{K(s+3)}{s^4 + 15s^3 + 60s^2 + (20+K)s + (-96+3K)}$$

$$s^4 \quad | \quad , \quad 60 \quad -96 + 3K$$

$$s^3 \quad 15 \quad | \quad 20+K$$

$$|$$

$$880-K > 0 \\ K < 880$$

$$\begin{aligned} s^2 & \left| \begin{array}{cc} 1 & 60 \\ 15 & 20+K \end{array} \right| = \frac{20+K-900}{-15} = \frac{880-K}{15}, \quad -96+3K \\ s^1 & \left| \begin{array}{cc} 15 & 20+K \\ 880-K & 45K-1440 \end{array} \right| = \frac{675K-21600-17600-860K+K^2}{-(880-K)} \\ s^0 & 45K-1440 \end{aligned}$$

$$= \frac{K^2 - 185K - 39200}{K-880}$$

$$K^2 - 185K - 39200 < 0 \quad \text{since } K-880 < 0$$

$$(K+126.03)(K-311.03) \\ -126.03 < K < 311.03$$

so

$$32 < K < 311.03$$

Marginal at $K=32$ ($w=0$)

$$K=311.03, \quad (880-311.03)s^2 + (45+311.03-1440)=0$$

$$568.97s^2 + 12556.75 = 0 \quad w = \sqrt{\frac{12556.75}{568.97}} = 4.7 \frac{\text{rad}}{\text{s}}$$

$$\text{IV) } G_{eq} = \frac{K}{(s+1)(s+10)} = \frac{K}{s^2 + 11s + 10}$$

$$T = \frac{G}{1+G} = \frac{K}{s^2 + 11s + 10 + K}$$

(a) From G_{eq} , Type O

$$(b) 10+K>0 \quad K > -\underline{10}$$

$$(c) K_p = \lim_{s \rightarrow 0} G(s) = \frac{K}{10}$$

$$(d) ess_{s \rightarrow \infty} = \frac{1}{1+K_p} = \frac{1}{1+10} = \frac{10}{10+K}$$

$$(e) \omega_n = \sqrt{10+K}$$

$$2\zeta\omega_n = 11 \quad T_s = \frac{8}{2\zeta\omega_n} = \frac{8}{11} = .727 \text{ s.}$$

does not depend on K — cannot make longer

$$(f) \zeta = \frac{-\ln(0.15)}{\sqrt{\pi^2 + \ln^2(0.15)}} = .517$$

$$2\zeta\omega_n = 11 \quad \omega_n = \frac{11}{2 \cdot .517} = 10.64$$

$$\sqrt{10+K} = 10.64 \quad K = 10.64^2 - 10 = \underline{103.17}$$

$$(g) .01 = \frac{10}{10+K} \quad .1 + .01K = 10 \quad K = \frac{9.9}{.01} = \underline{990}$$

$$\text{V) } T = \frac{K}{(s+2)(s^2+6s+10)} = \frac{K}{s^3 + 26s^2 + 130s + 200}$$

$$G_{\text{eg}} = \frac{T}{1-T} = \frac{K}{s^3 + 26s^2 + 130s + 200 - K}$$

(a) poles: $s = -20, -3 \pm j$ ($s^2 + 6s + 10 = (s+3)^2 + (1)^2$)
 zeros: None

(b) Using dominant \approx order polly,

$$w_n = \sqrt{10} = \sqrt{162} \quad \beta = \frac{6}{2\sqrt{10}} = .949 = \frac{3\sqrt{10}}{10}$$

$$\%OS = \exp(-\beta\pi/\sqrt{1-\beta^2}) = .00807\%$$

$$T_s = \frac{4}{\beta w_n} = \frac{8}{2\beta w_n} = \frac{4}{3} = 1.3 \text{ s.}$$

$$T_p = \frac{\pi}{w_n \sqrt{1-\beta^2}} = \frac{\pi}{\sqrt{10} \sqrt{1-\frac{72}{100}}} = \frac{\pi}{1} = \pi \text{ s.}$$

$$Tr \text{ from formula} = .98447 \text{ sec}$$

Accurate except for Tr since formula specifies $\beta < .9$

$$(c) \begin{array}{lll} K=200 & \text{Type I} \\ \text{otherwise} & \text{Type O} \end{array}$$

$$(d) K=200 \quad K_v = \lim_{s \rightarrow 0} s \frac{200}{s^3 + 26s^2 + 130s} = \frac{200}{130} = 1.54$$

BUT marginally stable! ess, ramp = $\frac{1}{K_v} = \frac{130}{200} = .65$

$$\text{otherwise } K_p = \lim_{s \rightarrow 0} \frac{K}{s^3 + 26s^2 + 130s + 200 - K} = \frac{K}{200 - K}$$

$$\text{ess, step} = \frac{1}{1+K_p} = \frac{1}{1+\frac{K}{200-K}} = \frac{200-K}{200}$$

VI

$$G_{eq} = \frac{K(s+1)}{s^2(s^2 + 14s + 58)} = \frac{Ks + K}{s^4 + 14s^3 + 58s^2}$$

$$T = \frac{G_{eq}}{1 - G_{eq}} = \frac{Ks + K}{s^4 + 14s^3 + 58s^2 + Ks + K}$$

$$(1) \quad s^4 \quad | \quad \begin{matrix} 1 \\ & 1 \\ & & 1 \end{matrix} \quad s^8 \quad K$$

$$s^3 \quad 14 \quad | \quad K$$

$$s^2 \quad \left| \begin{array}{cc} 1 & s8 \\ 14 & K \\ -14 & \end{array} \right| = \frac{K}{14} + s8 = \frac{812 - K}{14} \quad | \quad 14K$$

$$812 - K > 0 \\ K < 812$$

$$K^2 - 616K < 0 \\ \text{Since } K - 812 < 0$$

$$(K)(K - 616) < 0 \\ 0 < K < 616$$

$$s^1 \quad \left| \begin{array}{cc} 14 & K \\ 812 - K & 14K \\ K - 812 & \end{array} \right| = \frac{196K - 812K + K^2}{K - 812} = \frac{K^2 - 616K}{K - 812}$$

$$s^0 \quad \left| \begin{array}{c} 14K \\ K > 0 \\ \end{array} \right. \quad |$$

$$\underline{\text{So}} \quad \underline{0 < K < 616}$$

(b) Type II

$$(c) K_a = \lim_{s \rightarrow 0} s^2 \left(\frac{K(s+1)}{s^2(s^2 + 14s + 58)} \right) = \frac{K}{58}$$

$$R_{ss, para} = \frac{1}{K_a} = \frac{58}{K}$$

$$(d) \quad \omega s = \frac{58}{K} \quad K = 232$$

$$(e) \quad K = 616 \rightarrow (812 - 616)s^2 + 14(616) = 0$$

$$196s^2 + 8624 = 0$$

$$\omega = \sqrt{\frac{8624}{196}} = \underline{6.633 \text{ rad/s}}$$