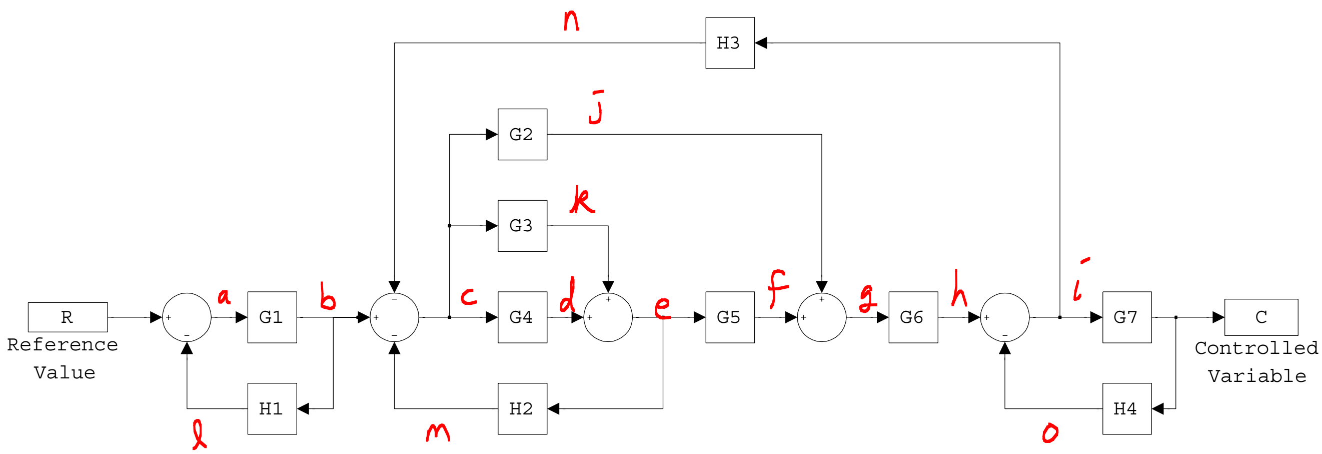
Name (please print):
Community Standard (print ACPUB ID):

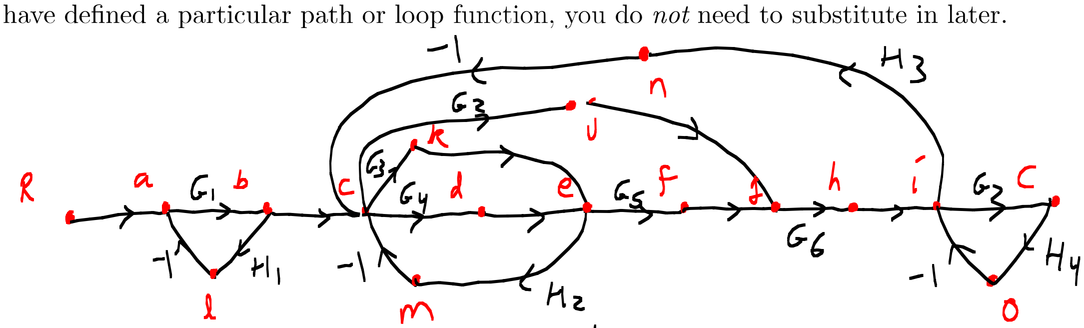
Problem I: [15 pts.] System Simplification

Given the system below:



(a) Clearly draw a signal flow diagram for the system. Be sure to indicate where each node is on the system.

(b) Use Mason's Rule to determine the overall transfer function $T(s) = C(s)/R(s)$. Remember that once you have defined a particular path or loop function, you do *not* need to substitute in later.



T:

T_1 Rabcdefghi C $= G_1G_4G_5G_7$

T_2 Rabcjghi C $= G_1G_2G_6G_7$

T_3 Rabckefghi C $= G_1G_3G_5G_6G_7$

Loops

L_1: abla $-G_1H_1$

L_2: cdemc $-G_4H_2$

L_3: ckemc $-G_3H_2$

L_4: cdefghinc $-G_4G_5G_6H_3$

L_5: ckefghinc $-G_3G_5G_6H_3$

L_6: cjghinc $-G_2G_6H_3$

L_7: iCoi $-G_7H_4$

$$\Delta = 1 - L_1 - L_2 - L_3 - L_4 - L_5 - L_6 - L_7 + L_1(L_2+L_3+L_4+L_5+L_6+L_7) + (L_2+L_3)L_7 - L_1L_2L_7 - L_1L_3L_7$$

all $\Delta_i = 1$

$$T = \frac{\Sigma T_i \Delta_i}{\Delta}$$

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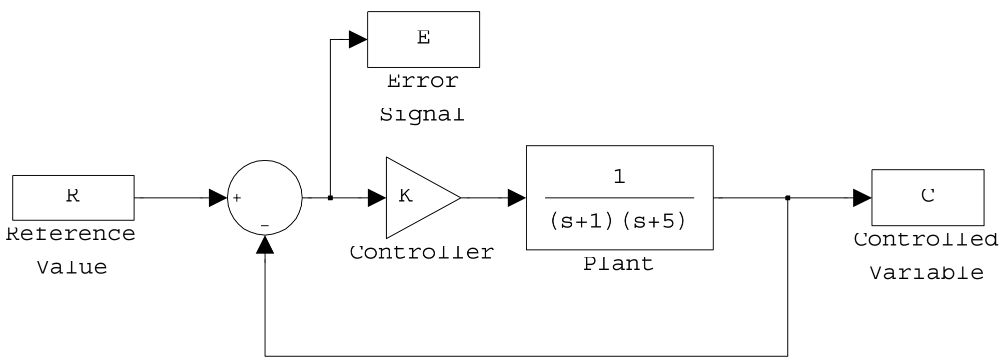
Problem II: [30 pts.] System Design I

This problem works with a system that has the following block diagram:



For this system:

(a) Determine the overall transfer function $T = \frac{C}{R}$
(b) Determine the values of K that keep the system stable. Be sure to provide supporting documentation.
(c) Is there a value of K for which the system is marginally stable? If there is, find it and also find the frequency of oscillation for the marginally stable system.
(d) For what value of K is the system critically damped?
(e) For what values of K is the system underdamped?
(f) Assuming K is such that the system is underdamped, determine ω_n and ζ as functions of K.
(g) The system is found to be underdamped when $K = 10$. Determine where the poles are for this value of K.
(h) Calculate estimates for the rise time, peak time, %OS, and settling time when $K = 10$. How much confidence do you have in your predictions, and why?
(i) Assuming $K > 0$: with respect to steady state error, what type of system is this, and why?
(j) Assuming $K > 0$: which is the appropriate finite static error constant, and what is its value as a function of K?
(k) Assuming $K > 0$: what is the steady state value of the error signal e, as a function of K, when the input is:
   (a) $r(t) = 10\,u(t)$
   (b) $r(t) = 10\,t\,u(t)$
   (c) $r(t) = 10\,t^2\,u(t)$

a) $T = \frac{C}{R} = \frac{G}{1+GH}$ $\quad G = \frac{K}{s^2+6s+5}$ $\quad H = 1$ $\quad T = \boxed{\frac{K}{s^2+6s+5+K}}$

b)
$$\begin{array}{lll} s^2 & 1 & 5+K \\ s^1 & 6 & \\ s^0 & 5+K & \end{array}$$
$\boxed{K > -5 \text{ for stability}}$

c) Marginally stable if $K = -5$; $6s = 0$ so $\boxed{\omega = 0}$

d) $\omega_n = \sqrt{5+K}$ $\quad 2\zeta\omega_n = 6$ so $\zeta = \frac{3}{\sqrt{5+K}} = 1$ if $\boxed{K = 4}$

e) Underdamped if $\zeta < 1$ so $K > 4$

f) As above, $\omega_n = \sqrt{5+K}$ $\quad \zeta = \dfrac{3}{\sqrt{5+K}}$

g) K=0, roots of $s^2+6s+15$

$$(s+3)^2 + (\sqrt{6})^2 \qquad s = -3 \pm j\sqrt{6}$$

h) K=10, $\omega_n = \sqrt{15} = 3.873$ $\quad \zeta = \dfrac{3}{\sqrt{15}} = .775$

$$T_{r,n} = 2.126 + \left(\frac{.775-.7}{.8-.7}\right)(2.467-2.126) = 2.382$$

$$T_r = \frac{T_{r,n}}{\omega_n} = \frac{2.382}{3.873} = \boxed{0.615}$$

$$T_p = \frac{\pi}{\omega_n\sqrt{1-\zeta^2}} = \boxed{1.284}$$

$$\%OS = \exp(-\zeta\pi/\sqrt{1-\zeta^2}) = \boxed{2.122}$$

$$T_s = \frac{4}{\zeta\omega_n} = \boxed{1.333}$$

Since this is a pure 2nd order system, lots of confidence.

i) $G = \dfrac{K}{(s+1)(s+5)}$ Type 0 system : no integrator

j) $K_p = \lim_{s\to 0} G = \dfrac{K}{5}$

k) $e_{ss,\,step} = 10 \times \dfrac{1}{1+K_p} = \dfrac{50}{5+K}$

(b), (c) ∞

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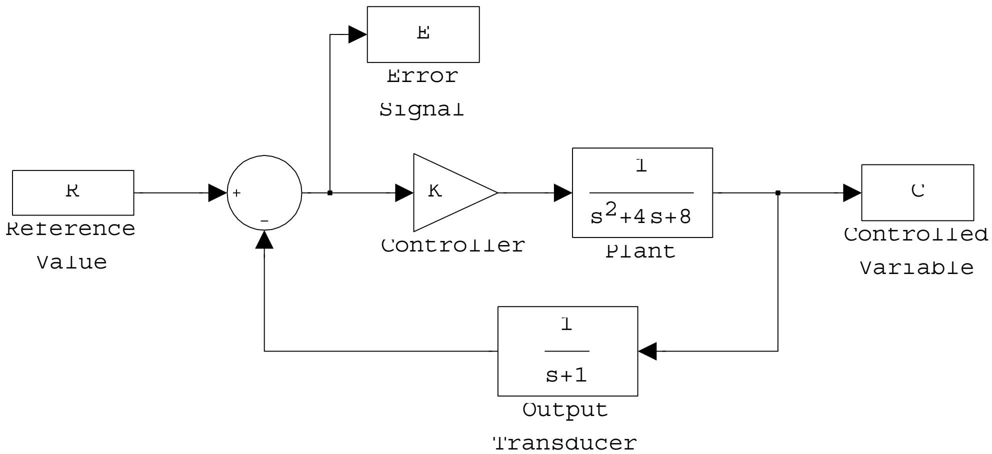
Problem III: [25 pts.] System Design II

This problem works with a system that has the following block diagram:



For this system:

(a) Determine the overall transfer function $T = \frac{C}{R}$

(b) Determine the values of K that keep the system stable. Be sure to provide supporting documentation.

(c) Is there a value of K for which the system is marginally stable? If there is, find it and also find the frequency of oscillation for the marginally stable system.

(d) The system is found to be stable when $K = 15$. Determine where the poles are for this value of K, then predict the peak time and %OS for this particular gain. How much confidence do you have in your predictions, and why?

(e) With respect to steady state error: what type of system is this, and why?

(f) What is the appropriate finite static error constant, and what is its value as a function of K?

(a) $G = \dfrac{K}{s^2+4s+8}$ $\quad H = \dfrac{1}{s+1}$ $\quad T = \dfrac{G}{1+GH} = \dfrac{K(s+1)}{(s^2+4s+8)(s+1)+K}$

$$T = \frac{K(s+1)}{s^3+5s^2+12s+8+K}$$

(b)

s^3	1	12
s^2	5	8+K
s^1	$\frac{60-8-K}{5}$	
s^0	8+K	

$K < 52$

$K > -8$

(c) Marginal at $K=8$; $\frac{48}{5}s=0$ so $\omega = 0$

Marginal at $K=52$; $5s^2+60=0$ so $\omega = \sqrt{12} = 3.464$

(d) $K=15$, $s^3+5s^2+12s+23=0$

from Matlab, poles at -3.4537
$-.7732 \pm j2.4621$

$$T_p = \frac{\pi}{\omega_d} = \frac{\pi}{2.4621} = \boxed{1.276}$$

$$\%OS = \exp(-\zeta\pi/\sqrt{1-\zeta^2}) = \exp(-\pi \times 0.7732/2.4621) = 37.3\%$$

Not much → zero at -1 is too close based on ω_d of 2.46 ...

(e) $$T = \frac{K(s+1)}{s^3+5s^2+12s+8+K} \qquad G_{eq} = \frac{T}{1-T}$$

$$G_{eq} = \frac{K(s+1)}{s^3+5s^2+12s+8+K-Ks-K} = \frac{K(s+1)}{s^3+5s^2+(12-K)s+8}$$

Type 0

(f) $$K_p = \lim_{s\to 0} G_{eq} = \boxed{\frac{K}{8}}$$

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Problem IV: [30 pts.] System Design III

A system with gain control K in the forward path is found to have an overall transfer function of:

$$T = \frac{K(s^2 - 4s + 3)}{s^3 + 8s^2 + 15s + Ks^2 - 4Ks + 3K}$$

For this system:

(a) Determine the values of K that keep the system stable. Be sure to provide supporting documentation.

(b) Is there a value of K for which the system is marginally stable? If there is, find it and also find the frequency of oscillation for the marginally stable system.

(c) With respect to steady state error: what type of system is this, and why?

(d) What is the appropriate finite static error constant, and what is its value as a function of K?

(e) What is the steady state value of the error signal e when the input is:

(a) $r(t) = 3\,u(t)$
(b) $r(t) = 3\,t\,u(t)$
(c) $r(t) = 3\,t^2\,u(t)$

(f) The system is shown to be stable when $K = 2$. Determine where the poles are for this value of K, then predict the peak time and %OS for this particular gain. How much confidence do you have in your predictions, and why?

(g) Use MATLAB to simulate the output of this system for 20 seconds when the gain is 2 and the input is a unit step function. Determine the simulated peak time and %OS graphically and report this on the test. Save the code for this to a file called `Sys3step.m` and save the plot as a PostScript file using the command:

```
print -deps Sys3StepPlot
```

(h) Use MATLAB to simulate the output of this system for 20 seconds when the gain is 2 and the input is a unit ramp ($r(t) = t\,u(t)$) function. Save the code for this to a file called `Sys3ramp.m` and save the plot as a PostScript file using the command:

```
print -deps Sys3RampPlot
```

(a) See Routh Array
$0 < K < 3.52$

(b) $K = 0, \quad w = 0$
$K = 3.52, \quad (K+8)s^2 + 3K = 0$
$11.52\,s^2 + 10.56 = 0 \qquad w = 0.957$

(c) $G_{eq} = \frac{T}{1-T} = \frac{K(s^2-4s+3)}{s^3+8s^2+15s+Ks^2-4Ks+3K-Ks^2+4Ks-3K} = \frac{K(s^2-4s+3)}{s^3+8s^2+15s}$

Type I system: $G_{eq} = \frac{K(s^2-4s+3)}{(s)(s^2+8s+15)}$

(d) $K_v = \lim_{s \to 0} s\,G_{eq} = \frac{3K}{15} = \frac{K}{5}$

(e) (a) $e_{ss,\,step} = 0$
(b) $e_{ss} = 3 \times \frac{1}{K_v} = \frac{15}{K}$
(c) $e_{ss,\,para} = \infty$

(f) $K=2$, $T = \dfrac{2(s^2-4s+3)}{s^3+10s^2+7s+6}$

zeros at $+1$, $+3$

poles at -9.318, $-.3411 \pm j\,.7264$

$T_p = \dfrac{\pi}{\omega_d} = \dfrac{\pi}{.7264} = 4.325$

$\%OS = \exp(-\pi\zeta/\sqrt{1-\zeta^2}) = \exp(-\pi \cdot .3411/.7264) = 22.87\%$

No Confidence → zeros in RHP!

(g+h) See Matlab

Code based on routh_hurwitz.mws by Stephen Bruder
Found on the web 3/31/2005 at
http://www.ee.nmt.edu/~bruder/ee443/handouts

Code last revised on 2/20/2008 by Michael Gustafson

```
> restart
```

$$> T := \frac{(K\cdot(s^2 - 4\cdot s + 3))}{s^3 + 8\cdot s^2 + 15\cdot s + K\cdot s^2 - 4\cdot K\cdot s + 3\cdot K}$$

$$T := \frac{K\,(s^2 - 4\,s + 3)}{s^3 + 8\,s^2 + 15\,s + K\,s^2 - 4\,K\,s + 3\,K} \quad \textbf{(1)}$$

$$> Char\_Eqn := \mathrm{denom}(T);$$

$$Char\_Eqn := s^3 + 8\,s^2 + 15\,s + K\,s^2 - 4\,K\,s + 3\,K \quad \textbf{(2)}$$

```
> Char_Eqn := sort(collect(simplify(Char_Eqn),s), s);
```

$$Char\_Eqn := s^3 + (K + 8)\,s^2 + (-4\,K + 15)\,s + 3\,K \quad \textbf{(3)}$$

```
> if epsflag<>0 then evalm(RT); subs(epsilon=1e-15,evalm(RT))
  else evalm(RT) end if;
```

$$\begin{bmatrix} s^3 & 1 & -4\,K + 15 \\ s^2 & K + 8 & 3\,K \\ s & -\dfrac{4\,(K^2 + 5\,K - 30)}{K + 8} & 0 \\ 1 & 3\,K & 0 \end{bmatrix} \quad \textbf{(4)}$$

The code below determines inequalities assuming some variable K.
If there are other variables, change the assume and solve lines.

```
> eqn:={};
  for i from 1 by 1 to n+1 do
       eqn:=eqn union {RT[i, 2]>0};
  od;
> assume(K, real);
  MyAns:=solve(eqn, K);
```

$$eqn := \{\ \}$$

$$eqn := \{0 < 1\}$$

$$eqn := \{0 < K + 8,\ 0 < 1\}$$

$$eqn := \left\{0 < K + 8,\ 0 < -\frac{4\,(K^2 + 5\,K - 30)}{K + 8},\ 0 < 1\right\}$$

$$eqn := \left\{0 < K + 8,\ 0 < -\frac{4\,(K^2 + 5\,K - 30)}{K + 8},\ 0 < 3\,K,\ 0 < 1\right\}$$

$$MyAns := \{0 < K\sim, K\sim < \mathrm{RootOf}(\_Z^2 + 5\,\_Z - 30, index = 1)\} \quad \textbf{(5)}$$

```
> evalf(MyAns);
```

$$\{K\sim < 3.520797289, 0. < K\sim\} \quad \textbf{(6)}$$

```
>
>
```

Problem 4

- Code for Step Input

```
clear; format short e

K = 2
T = K*tf([1 -4 3], [1 8+K 15-4*K 3*K])
step(T, 20)
```

- Code for Ramp Input

```
clear; format short e

K = 2
T = K*tf([1 -4 3], [1 8+K 15-4*K 3*K])
t = linspace(0, 20, 1000);
r = t.*(t>0);
c = lsim(T, r, t);

plot(t, r, 'k:', t, c, 'k-');
legend('Input', 'Output', 0);
title('Ramp Response')
xlabel('Time')
grid on

print -deps Sys3RampPlot
```

- Graphs for Step and Ramp Inputs