

Test | Spring 2008

Note Title

2/13/2008

$$M_1: ((M_1)s^2 + (f_{v1} + f_{v3})s + (K_1 + K_3 + K_4))X_1 - (K_1)X_2 - (K_4)X_3 = 0$$

$$M_2: -K_1X_1 + (M_2s^2 + f_{v2}s + K_1 + K_2)X_2 - (f_{v2}s + K_2)X_4 = 0$$

$$M_3: -K_1X_1 + (M_3s^2 + f_{v4}s + K_4 + K_5)X_3 - (f_{v4}s + K_5)X_4 = 0$$

$$M_4: -(f_{v2}s + K_2)X_2 - (f_{v4}s + K_5)X_3 + (M_4s^2 + (f_{v2} + f_{v4} + f_{v3})s + K_2 + K_5)X_4 = F$$

(b) See Maple

Problem I - ECE 141 Test 1 - Spring 2008

> *restart*;

> $eqn1 := (M1 \cdot s^2 + (fv1 + fv3) \cdot s + (K1 + K3 + K4)) \cdot X1 - K1 \cdot X2 - K4 \cdot X3 = 0 :$
 > $eqn2 := -K1 \cdot X1 + (M2 \cdot s^2 + fv2 \cdot s + K1 + K2) \cdot X2 - (fv2 \cdot s + K2) \cdot X4 = 0 :$
 > $eqn3 := -K4 \cdot X1 + (M3 \cdot s^2 + fv4 \cdot s + K4 + K5) \cdot X3 - (fv4 \cdot s + K5) \cdot X4 = 0 :$
 > $eqn4 := -(fv2 \cdot s + K2) \cdot X2 - (fv4 \cdot s + K5) \cdot X3 + (M4 \cdot s^2 + (fv2 + fv4 + fv5) \cdot s + K2 + K5) \cdot X4 = F :$

> $MyVals := M1 = 1, M2 = 1, M3 = 1, M4 = 1, fv1 = 1, fv2 = 1, fv3 = 1, fv4 = 1, fv5 = 1, K1 = 1, K2 = 1, K3 = 1, K4 = 1, K5 = 1 :$

> $MySoln := sort(simplify(subs(MyVals, solve(\{eqn1, eqn2, eqn3, eqn4\}, [X1, X2, X3, X4]))), s), s :$

> *assign*(*MySoln*)

> $sort(X1, s); sort(simplify(expand(X1)), s)$

$$\frac{F(2s^3 + 4s^2 + 6s + 4)}{s^8 + 7s^7 + 24s^6 + 54s^5 + 81s^4 + 85s^3 + 58s^2 + 22s + 4} \quad (1)$$

$$\frac{2F}{s^5 + 5s^4 + 11s^3 + 15s^2 + 8s + 2}$$

> $sort(X2, s); sort(simplify(expand(X2)), s)$

$$\frac{(s^5 + 4s^4 + 10s^3 + 14s^2 + 13s + 6)F}{s^8 + 7s^7 + 24s^6 + 54s^5 + 81s^4 + 85s^3 + 58s^2 + 22s + 4} \quad (2)$$

$$\frac{(s^2 + 2s + 3)F}{s^5 + 5s^4 + 11s^3 + 15s^2 + 8s + 2}$$

> $sort(X3, s); sort(simplify(expand(X3)), s)$

$$\frac{(s^5 + 4s^4 + 10s^3 + 14s^2 + 13s + 6)F}{s^8 + 7s^7 + 24s^6 + 54s^5 + 81s^4 + 85s^3 + 58s^2 + 22s + 4} \quad (3)$$

$$\frac{(s^2 + 2s + 3)F}{s^5 + 5s^4 + 11s^3 + 15s^2 + 8s + 2}$$

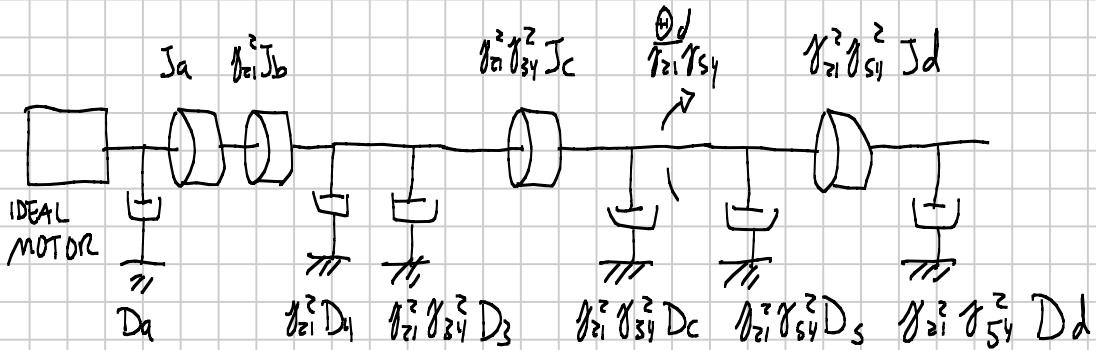
> $sort(X4, s); sort(simplify(expand(X4)), s)$

$$\frac{F(s^6 + 4s^5 + 12s^4 + 20s^3 + 25s^2 + 18s + 8)}{s^8 + 7s^7 + 24s^6 + 54s^5 + 81s^4 + 85s^3 + 58s^2 + 22s + 4} \quad (4)$$

$$\frac{(s^4 + 3s^3 + 7s^2 + 7s + 4)F}{s^6 + 6s^5 + 16s^4 + 26s^3 + 23s^2 + 10s + 2}$$

>

II)



$$J_{eq} = J_a + \gamma_{z1}^2 (J_b + \gamma_{34}^2 J_c + \gamma_{54}^2 J_d)$$

$$D_{eq} = D_a + \gamma_{z1}^2 (D_4 + \gamma_{34}^2 (D_3 + D_c) + \gamma_{54}^2 (D_s + D_d))$$

$$\frac{H_a}{E_a} = \frac{\frac{1}{J_{eq}} \frac{K_t}{R_a}}{s(s + \frac{1}{J_{eq}} (D_{eq} + \frac{K_t K_b}{R_a}))}$$

$$H_d = \gamma_{z1} \gamma_{54} H_a$$

$$G(s) = \frac{H_d}{E_a} = \frac{\frac{\gamma_{z1} \gamma_{54}}{J_{eq}} \frac{K_t}{R_a}}{s(s + \frac{1}{J_{eq}} (D_{eq} + \frac{K_t K_b}{R_a}))}$$

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> *restart*

$$\begin{aligned} > \text{Jeqn} := Ja + \gamma 21^2 \cdot (Jb + \gamma 34^2 \cdot Jc + \gamma 54^2 \cdot Jd) \\ &\quad \text{Jeqn} := Ja + \gamma 21^2 (Jb + \gamma 34^2 Jc + \gamma 54^2 Jd) \end{aligned} \quad (1)$$

$$\begin{aligned} > \text{Deqn} := Da + \gamma 21^2 \cdot (D4 + \gamma 34^2 \cdot (D3 + Dc) + \gamma 54^2 \cdot (D5 + Dd)) \\ &\quad \text{Deqn} := Da + \gamma 21^2 (D4 + \gamma 34^2 (D3 + Dc) + \gamma 54^2 (D5 + Dd)) \end{aligned} \quad (2)$$

$$\begin{aligned} > \text{MyVals} := Ja = 1, Jb = 200, Jc = 10000, Jd = 10000, Da = 1, Dc = 4000, Dd \\ &\quad = 9000, D3 = 1000, D4 = 100, D5 = 1000, Kb = \frac{6 \cdot Ra}{Kt}, Kt = 10 \cdot Ra \\ &\quad \text{MyVals} := Ja = 1, Jb = 200, Jc = 10000, Jd = 10000, Da = 1, Dc = 4000, Dd \\ &\quad = 9000, D3 = 1000, D4 = 100, D5 = 1000, Kb = \frac{6 Ra}{Kt}, Kt = 10 Ra \end{aligned} \quad (3)$$

$$\begin{aligned} > \text{MyGears} := \gamma 21 = \frac{N1}{N2}, \gamma 34 = \frac{N4}{N3}, \gamma 54 = \frac{N4}{N5}, N1 = 2, N2 = 20, N3 = 50, N4 = 10, \\ &\quad N5 = 100 \\ &\quad \text{MyGears} := \gamma 21 = \frac{N1}{N2}, \gamma 34 = \frac{N4}{N3}, \gamma 54 = \frac{N4}{N5}, N1 = 2, N2 = 20, N3 = 50, N4 = 10, \\ &\quad N5 = 100 \end{aligned} \quad (4)$$

$$\begin{aligned} > \text{subs}(\text{MyGears}, \text{MyVals}, \text{Jeqn}) \quad 8 \end{aligned} \quad (5)$$

$$\begin{aligned} > \text{subs}(\text{MyGears}, \text{MyVals}, \text{Deqn}) \quad 5 \end{aligned} \quad (6)$$

$$\begin{aligned} > G := \frac{N4}{N5} \cdot \frac{N1}{N2} \cdot \frac{\left(\frac{1}{\text{Jeqn}} \cdot \frac{Kt}{Ra} \right)}{s \cdot \left(s + \frac{1}{\text{Jeqn}} \cdot \left(\text{Deqn} + \frac{Kt \cdot Kb}{Ra} \right) \right)} : \\ > \text{collect}(\text{simplify}(\text{subs}(\text{MyVals}, \text{MyGears}, G)), s) \\ &\quad \frac{1}{10 s (8 s + 11)} \end{aligned} \quad (7)$$

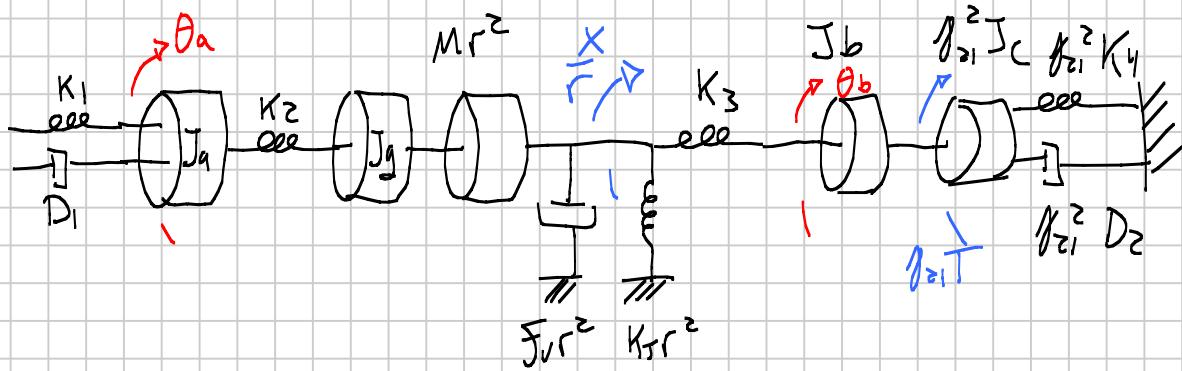
$$\begin{aligned} > \text{with}(\text{inttrans}) \\ &[\text{addtable}, \text{fourier}, \text{fouriercos}, \text{fouriersin}, \text{hankel}, \text{hilbert}, \text{invfourier}, \text{invhilbert}, \\ &\quad \text{invlaplace}, \text{invmellin}, \text{laplace}, \text{mellin}, \text{savetable}] \end{aligned} \quad (8)$$

$$\begin{aligned} > \text{invlaplace}(\text{subs}(\text{MyVals}, \text{MyGears}, G), s, t) \\ &\quad \frac{1}{110} - \frac{1}{110} e^{-\frac{11}{8} t} \end{aligned} \quad (9)$$

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III)

a)

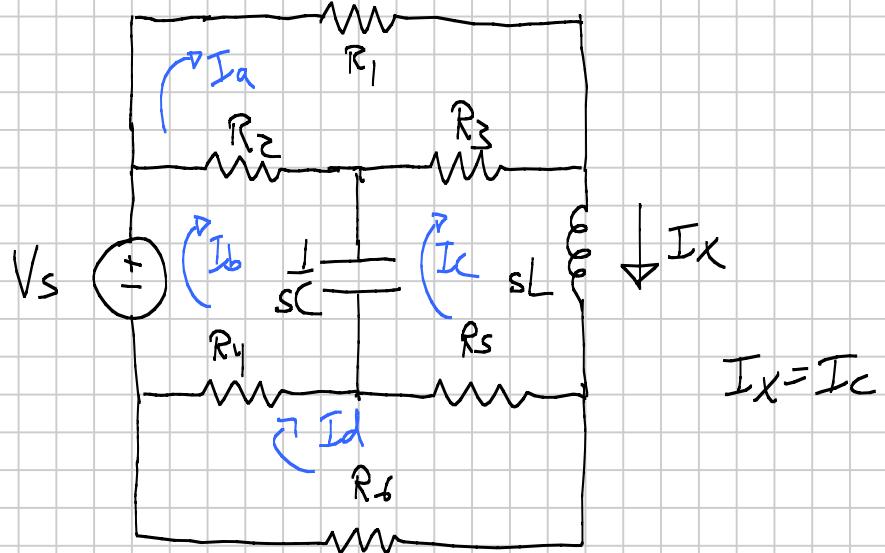


$$1: (J_a s^2 + D_1 s + (K_1 + K_2)) \dot{\theta}_a - (K_2) (\ddot{x}/r) = 0$$

$$2: -(K_2) \dot{\theta}_a + ((J_b + M r^2) s^2 + (\gamma_{z1} r^2) s + (K_2 + K_3 + K_T r^2)) (\ddot{x}/r) - K_3 \dot{\theta}_b = 0$$

$$3: -K_3 (\ddot{x}/r) + ((J_b + \gamma_{z1}^2 J_c) s^2 + \gamma_{z1}^2 D_2 s + K_2 + \gamma_{z1}^2 K_4) \dot{\theta}_b = \gamma_{z1} T$$

IV



$$I_x = I_c$$

L a) $R_2(I_a - I_b) + R_1(I_a) + R_3(I_a - I_c) = 0$

L b) $-V_s + R_2(I_b - I_a) + \frac{1}{sC}(I_b - I_c) + R_4(I_b - I_d) = 0$

L c) $\frac{1}{sC}(I_c - I_b) + R_3(I_c - I_a) + sL(I_c) + R_s(I_c - I_d) = 0$

L d) $R_4(I_d - I_b) + R_s(I_d - I_c) + R_6(I_d) = 0$

b)-d) See Maple

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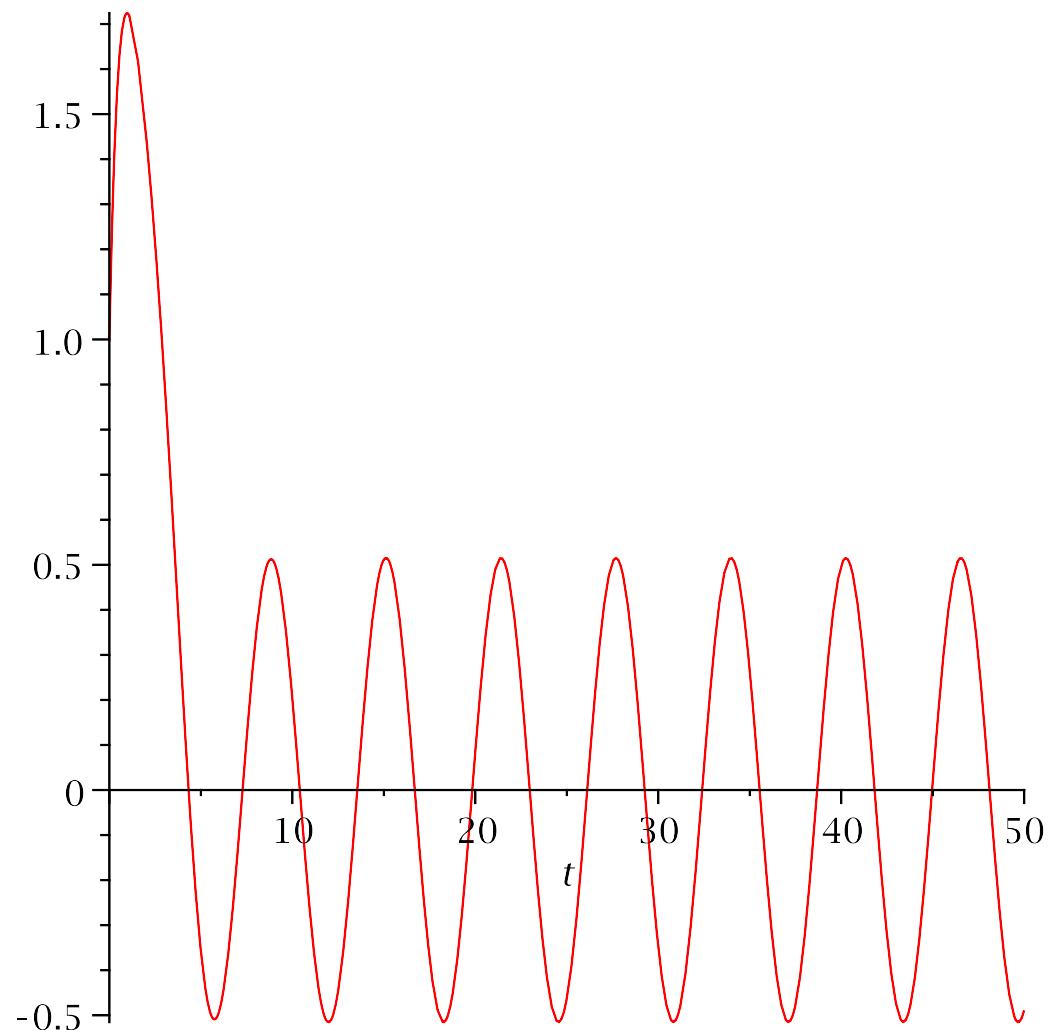
> restart
> eqn1 := R2·(Ia - Ib) + R1·Ia + R3·(Ia - Ic) = 0 :
> eqn2 := -Vs + R2·(Ib - Ia) +  $\frac{1}{s·C}·(Ib - Ic) + R4·(Ib - Id) = 0 :$ 
> eqn3 :=  $\frac{1}{s·C}·(Ic - Ib) + R3·(Ic - Ia) + s·L·Ic + R5·(Ic - Id) = 0 :$ 
> eqn4 := R4·(Id - Ib) + R5·(Id - Ic) + R6·Id = 0 :
> MyVals := R1 = 1, R2 = 1, R3 = 1, R4 = 1, R5 = 1, R6 = 1, L = 1, C = 1 :
> MySoln := simplify(subs(MyVals, solve({eqn1, eqn2, eqn3, eqn4}, [Ia, Ib, Ic, Id])))
MySoln := 
$$\begin{bmatrix} Ia = \frac{(2 + 2s + s^2) Vs}{4 + 4s^2 + 7s}, Ib = \frac{Vs(3 + 4s + 3s^2)}{4 + 4s^2 + 7s}, Ic = \frac{(3 + 2s) Vs}{4 + 4s^2 + 7s}, \\ Id = \frac{(2 + 2s + s^2) Vs}{4 + 4s^2 + 7s} \end{bmatrix} \quad (1)$$

> assign(MySoln)
> Y := sort( $\left(\frac{Ic}{Vs}, s\right)$ 

$$Y := \frac{2s + 3}{4s^2 + 7s + 4} \quad (2)$$

> deqn1 := 4·diff(i(t), t$2) + 7·diff(i(t), t) + 4·i(t) = 2·diff(vs(t), t) + 3·vs(t)
deqn1 :=  $4\left(\frac{d^2}{dt^2} i(t)\right) + 7\left(\frac{d}{dt} i(t)\right) + 4 i(t) = 2\left(\frac{d}{dt} vs(t)\right) + 3 vs(t) \quad (3)$ 
> MyAnswer := simplify(dsolve({subs(vs(t) = sin(t), deqn1), i(0) = 1, D(i)(0) = 2}, {i(t)}))
MyAnswer :=  $i(t) = \frac{166}{105} e^{-\frac{7}{8}t} \sin\left(\frac{1}{8}\sqrt{15}t\right)\sqrt{15} + \frac{10}{7} e^{-\frac{7}{8}t} \cos\left(\frac{1}{8}\sqrt{15}t\right) - \frac{3}{7} \cos(t) + \frac{2}{7} \sin(t) \quad (4)$ 
> assign(MyAnswer)
> plot(i(t), t = 0 .. 50)

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