

Test 1 Spring 2007

Note Title

2/18/2007

$$I) \quad \frac{d^2x}{dt^2} + 8 \frac{dx}{dt} + 41x = (2t-2) u(t) \quad \begin{matrix} \dot{x}(0) = -1 \\ x(0) = 3 \end{matrix}$$

$$(s^2X - sx(0) - \dot{x}(0)) + 8(sX - x(0)) + 41X = \frac{2}{s^2} - \frac{2}{s}$$

$$s^2X - 3s + 1 + 8sX - 24 + 41X = \frac{-2s+2}{s^2}$$

$$(s^2 + 8s + 41)X = \frac{-2s+2}{s^2} + 3s + 23 = \frac{3s^3 + 23s^2 - 2s + 2}{s^2}$$

$$X = \frac{3s^3 + 23s^2 - 2s + 2}{(s^2 + 8s + 41)(s^2)} = \frac{A}{s^2} + \frac{B}{s} + \frac{C(s+4) + D(s)}{(s+4)^2 + (s)^2}$$

$\hookrightarrow s^2 + 8s + 16 + 2s = (s+4)^2 + s^2$

$$A = \lim_{s \rightarrow 0} s^2 X = \frac{2}{41} = .04878$$

$$\lim_{s \rightarrow -4+5j} (C(s+4) + D(s)) = \lim_{s \rightarrow -4+5j} (s^2 + 8s + 41)X$$

$$C(5j) + D(5) = \frac{3(-4+5j)^3 + 23(-4+5j)^2 - 2(-4+5j) + 2}{(-4+5j)^2} = \frac{511 - 585j}{-9 - 40j}$$

$$C(5j) + D(5) = 11.184 + 15.291j$$

$$C = 15.291/5 = 3.058 \quad D = 11.184/5 = 2.237$$

for B, more cross-multiplication yields a numerator of

$$A(s^2 + 8s + 41) + B(s)(s^2 + 8s + 41) + (C(s+4) + D(s))s^2 = 3s^3 + 23s^2 - 2s + 2$$

$$s^3 \text{ terms: } 0A + 1B + 1C + 0D = 3 \quad B = 3 - C = -0.058$$

$$X = \frac{.04878}{s^2} - \frac{0.058}{s} + \frac{3.058(s+4) + 2.237(s)}{(s+4)^2 + (s)^2}$$

$$x(t) = .04878t - .058 + e^{-4t}(3.058 \cos(5t) + 2.237 \sin(5t))$$

II) Assuming x_i = position of i^{th} block = x_5 is b/w $k_2 + f_{v4}$;

$$1) (M_1 s^2 + (f_{v1} + f_{v2} + f_{v3})s + (k_1 + k_2))X_1 - (f_{v2}s)X_2 - (f_{v3}s)X_3 - (k_3)X_5 = 0$$

$$2) -(f_{v2}s)X_1 + (M_2 s^2 + f_{v2}s + k_2)X_2 - (k_2)X_3 = 0$$

$$3) -(f_{v3}s)X_1 - (k_2)X_2 + (M_3 s^2 + f_{v3}s + k_2)X_3 = 0$$

$$4) (M_4 s^2 + (f_{v4} + f_{v5})s)X_4 - (f_{v4}s)X_5 = F$$

$$5) -(k_3)X_1 - (f_{v4}s)X_4 + (f_{v4}s + k_3)X_5 = 0$$

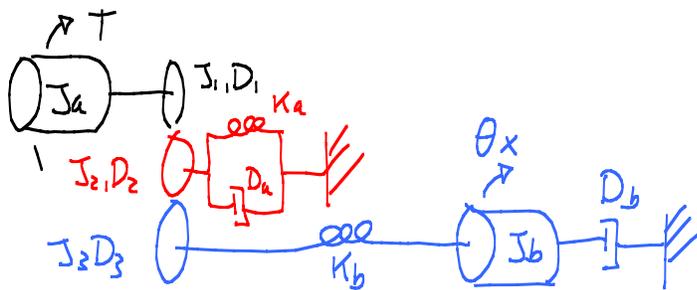
$$\begin{bmatrix} M_1 s^2 + (f_{v1} + f_{v2} + f_{v3})s + (k_1 + k_2) & -f_{v2}s & -f_{v3}s & 0 & -k_3 \\ -f_{v2}s & (M_2 s^2 + f_{v2}s + k_2) & -k_2 & 0 & 0 \\ -f_{v3}s & -k_2 & (M_3 s^2 + f_{v3}s + k_2) & 0 & 0 \\ 0 & 0 & 0 & (M_4 s^2 + (f_{v4} + f_{v5})s) & -f_{v4}s \\ -k_3 & 0 & 0 & -f_{v4}s & (f_{v4}s + k_3) \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \\ X_5 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ F \\ 0 \end{bmatrix}$$

Check: adding columns will leave elements connected to ground or masses only

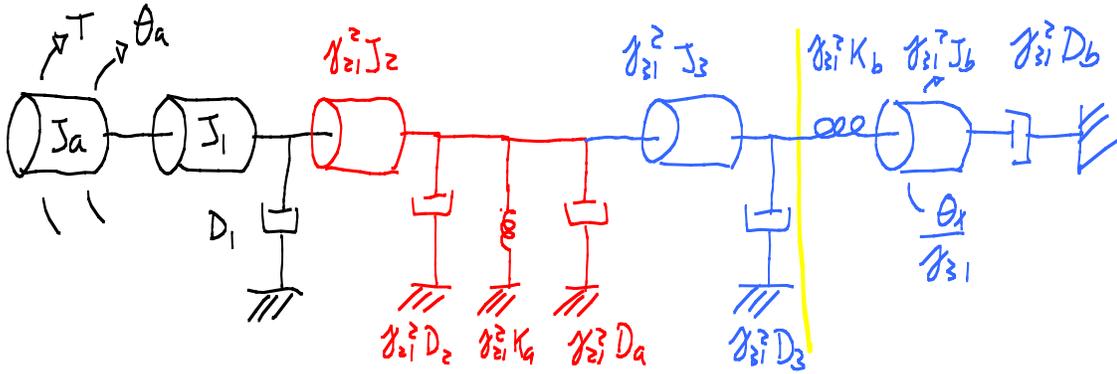
$$[M_1 s^2 + f_{v1}s + k_1 \quad M_2 s^2 \quad M_3 s^2 \quad M_4 s^2 + f_{v5}s \quad 0] \quad \checkmark$$

Check 2: w/ no gears or motors, matrix should have symmetry \checkmark

III)



Reflect up to gear 1. Note: $\gamma_{32}\gamma_{21} = \frac{N_2}{N_3} \frac{N_1}{N_2} = \frac{N_1}{N_3} = \gamma_{31}$ here.



$$J_x = J_a + J_1 + \gamma_{21}^2 J_2 + \gamma_{31}^2 J_3$$

$$D_x = D_1 + \gamma_{21}^2 D_2 + \gamma_{21}^2 D_a + \gamma_{31}^2 D_3$$

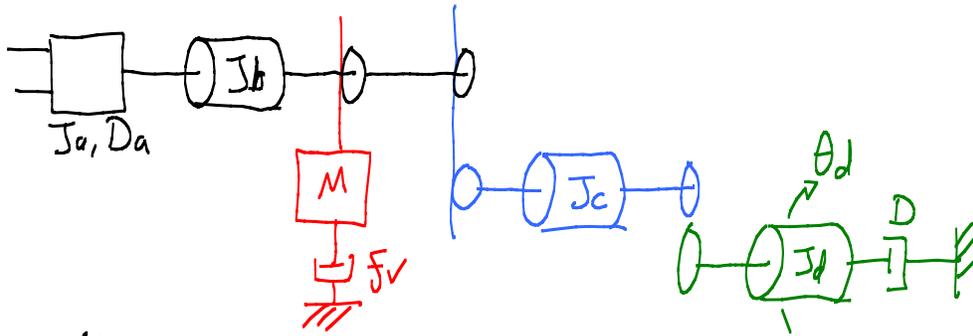
$$(J_x s^2 + D_x s + \gamma_{21}^2 K_a + \gamma_{31}^2 K_b) \Theta_a - (\gamma_{31}^2 K_b) \frac{\Theta_x}{\gamma_{31}} = T$$

$$- (\gamma_{31}^2 K_b) \Theta_a + (\gamma_{31}^2 J_b s^2 + \gamma_{31}^2 D_b s + \gamma_{31}^2 K_b) \frac{\Theta_x}{\gamma_{31}} = 0$$

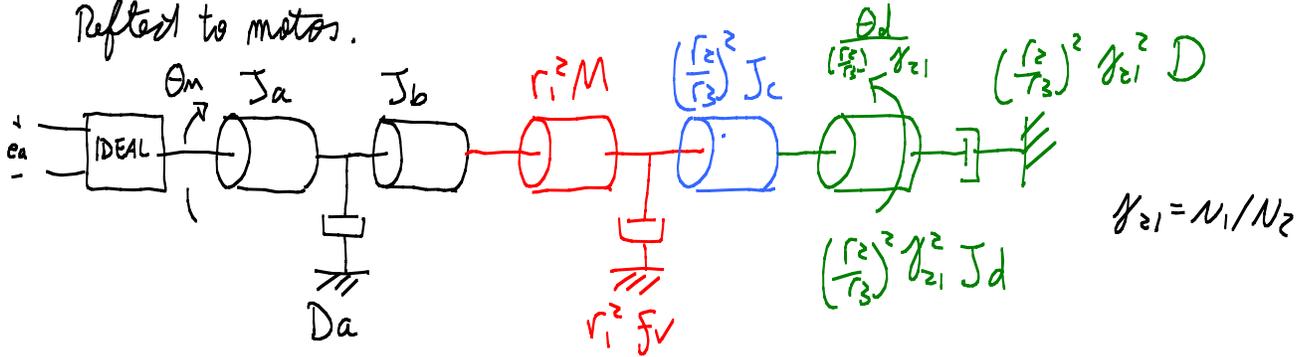
$$\begin{bmatrix} J_x s^2 + D_x s + \gamma_{21}^2 K_a + \gamma_{31}^2 K_b & -\gamma_{31} K_b \\ -\gamma_{31}^2 K_b & \gamma_{31} (J_b s^2 + D_b s + K_b) \end{bmatrix} \begin{bmatrix} \Theta_a \\ \Theta_x \end{bmatrix} = \begin{bmatrix} T \\ 0 \end{bmatrix}$$

Several possible correct answers.

IV)



Reflected to motor.



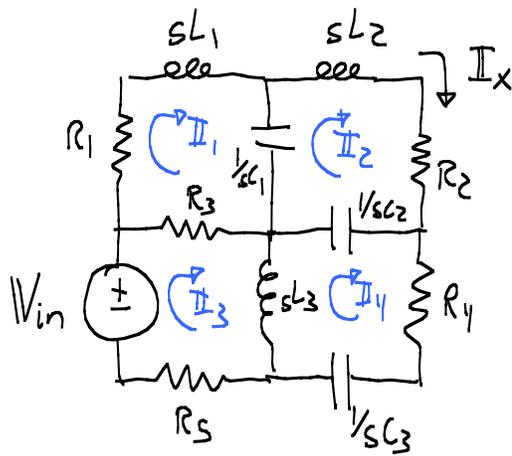
$$J_M = J_a + J_b + r_i^2 M + \left(\frac{r_2}{r_3}\right)^2 J_c + \left(\frac{r_2}{r_3}\right)^2 \gamma_{z1}^2 J_d$$

$$D_M = D_a + r_i^2 F_v + \left(\frac{r_2}{r_3}\right)^2 \gamma_{z1}^2 D$$

$$\frac{\Theta_M}{E_a} = \frac{\frac{1}{J_M} \frac{K_t}{R_a}}{s \left(s + \frac{1}{J_M} \left(D_M + \frac{K_t K_b}{R_a} \right) \right)} \quad \text{and} \quad \Theta_M = -\frac{\Theta_d}{\left(\frac{r_2}{r_3}\right) \gamma_{z1}} \quad \text{so}$$

$$\frac{\Theta_d}{E_a} = \frac{-\left(\frac{r_2}{r_3}\right) \gamma_{z1} \cdot \frac{1}{J_M} \frac{K_t}{R_a}}{s \left(s + \frac{1}{J_M} \left(D_M + \frac{K_t K_b}{R_a} \right) \right)}$$

V)



Note $I_2 = I_x$

$$1) (sL_1 + R_1 + R_3 + 1/sC_1)I_1 - (1/sC_1)I_2 - (R_3)I_3 - (0)I_4 = 0$$

$$2) -(1/sC_1)I_1 + (sL_2 + R_2 + 1/sC_1 + 1/sC_2)I_2 - (0)I_3 - (1/sC_2)I_4 = 0$$

$$3) -(R_3)I_1 - (0)I_2 + (sL_3 + R_3 + R_5)I_3 - (sL_3)I_4 = V_{in}$$

$$4) -(0)I_1 - (1/sC_2)I_2 - (sL_3)I_3 + (sL_3 + R_4 + 1/sC_2 + 1/sC_3)I_4 = 0$$

$$\begin{bmatrix} sL_1 + R_1 + R_3 + 1/sC_1 & -1/sC_1 & -R_3 & 0 \\ -1/sC_1 & sL_2 + R_2 + 1/sC_1 + 1/sC_2 & 0 & -1/sC_2 \\ -R_3 & 0 & sL_3 + R_3 + R_5 & -sL_3 \\ 0 & -1/sC_2 & -sL_3 & sL_3 + R_4 + 1/sC_2 + 1/sC_3 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ V_{in} \\ 0 \end{bmatrix}$$

Check: adding columns yield elements only on one loop.

$$: [sL_1 + R_1 \quad sL_2 + R_2 \quad R_5 \quad R_4 + 1/sC_3] \quad \checkmark$$

W/ no controlled sources or op-amps, matrix is symmetric \checkmark