

Problem 1

Note Title

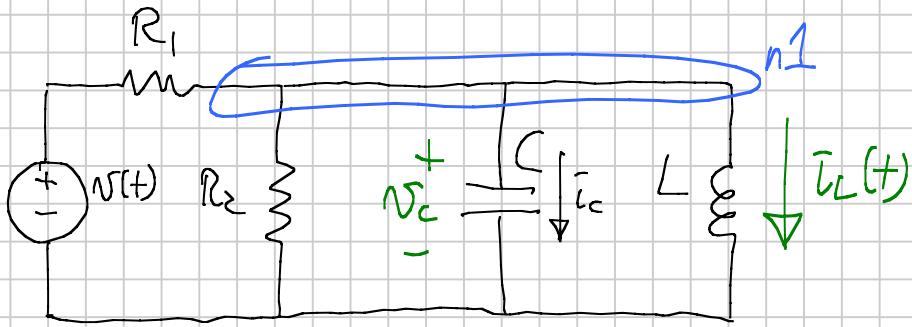
Use denominator to create table:

	s^7	1	-2	0	-9
L					
R					
Based on polynomial	$s^5 (+)$	$\begin{vmatrix} -2 \\ 2 \end{vmatrix} = -3$	$\begin{vmatrix} 1 & 0 \\ 2 & 6 \end{vmatrix} = -3$	$\begin{vmatrix} 1 & -9 \\ 2 & 0 \end{vmatrix} = -9$	All 0, so take derivative of prior row: $d(-3s^5 - 3s^3 - 9s) = -15s^4 - 9s^2 - 9$
(NOT R)	$s^4 (-)$	$\begin{vmatrix} 2 & 2 \\ -3 & -3 \end{vmatrix} = 0$	$\begin{vmatrix} 2 & 6 \\ -3 & -9 \end{vmatrix} = 0$	$\begin{vmatrix} 1 & -9 \\ -3 & 0 \end{vmatrix} = 0$	
(NOT R)	$s^3 (-)$	$\begin{vmatrix} -3 & -3 \\ -15 & -9 \end{vmatrix} = -18$	$\begin{vmatrix} -3 & -9 \\ -15 & -7 \end{vmatrix} = -108$	$\begin{vmatrix} 1 & -9 \\ -15 & 0 \end{vmatrix} = 15$	* Multiply by 15
R	$s^2 (+)$	$\begin{vmatrix} -15 & -9 \\ -18 & -108 \end{vmatrix} = \frac{1458}{18}$	$\begin{vmatrix} 1 & -9 \\ -18 & -108 \end{vmatrix} = \frac{-162}{18}$	$\begin{vmatrix} 1 & -9 \\ -15 & 0 \end{vmatrix} = 15$	* Multiply by 18
R	$s^1 (-)$	$\begin{vmatrix} -18 & -108 \\ 1458 & -162 \end{vmatrix} = -160380$	$\begin{vmatrix} 1 & -9 \\ -15 & 0 \end{vmatrix} = 15$		
(NOT R)	$s^0 (-)$	$\rightarrow 2s^2$			

Since last S must be symmetric, $2R \quad 2L \quad jw +$

$$\boxed{3R \quad 3L \quad jw}$$

Problem 2:



Use "typical" state variables, V_C and i_L , and get equations for their derivatives using capacitor - inductor voltage

$$(1) \quad V_L = L \frac{di_L}{dt} = V_C \text{ since they are in parallel}$$

$$(2) \quad \text{KCL at } n_1: \frac{V_C - V}{R_1} + \frac{V_C}{R_2} + C \frac{dV_C}{dt} + i_L = 0$$

Assuming $z_1 = i_L$, $z_2 = V_C$, $u = V$, $y = i_C$

$$(1) \quad \dot{z}_1 = 0 z_1 + \frac{1}{L} z_2 + 0 u$$

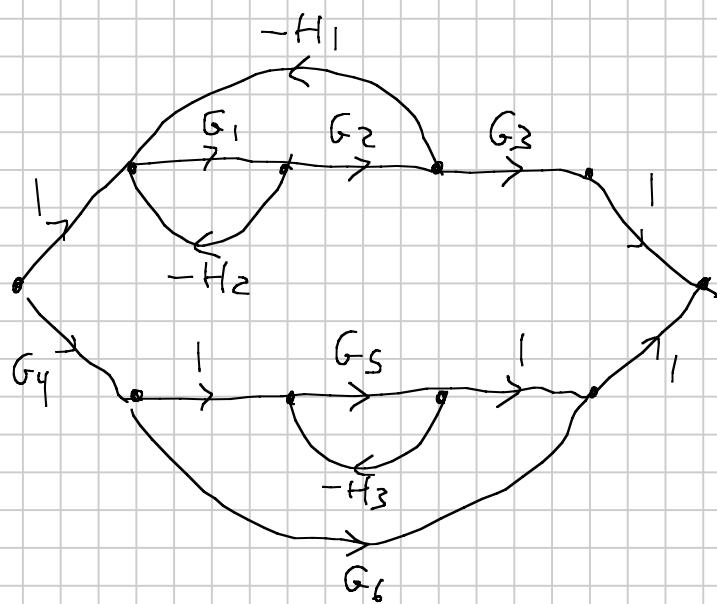
$$(2) \quad \dot{z}_2 = -\frac{1}{C} z_1 - \left(\frac{1}{R_1 C} + \frac{1}{R_2 C} \right) z_2 + \frac{1}{R_1 C} u$$

Note (n1) also gives $\frac{V_C - V}{R_1} + \frac{V_C}{R_2} + i_C + i_L = 0$

$$\text{so } y = -z_1 - \left(\frac{1}{R_1} + \frac{1}{R_2} \right) z_2 + \frac{1}{R_1} u$$

$$\begin{bmatrix} \dot{z}_1 \\ \dot{z}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1/L \\ -1/C & -(1/R_1 C + 1/R_2 C) \end{bmatrix} \begin{bmatrix} z_1 \\ z_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1/R_1 C \end{bmatrix} u$$

$$y = \begin{bmatrix} -1 & -(1/R_1 + 1/R_2) \end{bmatrix} \begin{bmatrix} z_1 \\ z_2 \end{bmatrix} + \begin{bmatrix} 1/R_1 \end{bmatrix} u$$



$$T = \frac{\sum T_i A_i}{\Delta}$$

$$T_1 = (I)(G_1)(G_2)(G_3)(I) = G_1 G_2 G_3$$

$$L_1 = (G_1)(-H_2) = -G_1 H_2$$

$$T_2 = (G_4)(I)(G_5)(I)(I) = G_4 G_5$$

$$L_2 = (G_1)(G_2)(-H_1) = -G_1 G_2 H_1$$

$$T_3 = (G_4)(G_6)(I) = G_4 G_6$$

$$L_3 = (G_5)(-H_3) = -G_5 H_3$$

$$\Delta = I - L_1 - L_2 - L_3 + L_1 L_3 + L_2 L_3 = I + G_1 H_2 + G_1 G_2 H_1 + G_5 H_3 + G_1 G_5 H_2 H_3 + G_1 G_2 G_5 H_1 H_3$$

$$\Delta_1 = I - L_3 = I + G_5 H_3$$

$$\Delta_2 = I - L_1 - L_2 = I + G_1 H_2 + G_1 G_2 H_1$$

$$\Delta_3 = I - L_1 - L_2 - L_3 + L_1 L_3 + L_2 L_3 = I + G_1 H_2 + G_1 G_2 H_1 + G_5 H_3 + G_1 G_5 H_2 H_3 + G_1 G_2 G_5 H_1 H_3$$

$$T = \frac{T_1 \Delta_1 + T_2 \Delta_2 + T_3 \Delta_3}{\Delta} =$$

$$\frac{(G_1 G_2 G_3)(I + G_5 H_3) + (G_4 G_5)(I + G_1 H_2 + G_1 G_2 H_1) + (G_4 G_6)(I + G_1 H_2 + G_1 G_2 H_1 + G_5 H_3 + G_1 G_5 H_2 H_3 + G_1 G_2 G_5 H_1 H_3)}{(I + G_1 H_2 + G_1 G_2 H_1 + G_5 H_3 + G_1 G_5 H_2 H_3 + G_1 G_2 G_5 H_1 H_3)}$$

Problem 4

$$G(s) = \frac{1}{s^4 + 4s^3 + 3s^2 + 4s + 4} \quad H(s) = K_1 s^2 + K_2$$

$$T = \frac{G}{1+GH} = \frac{1}{s^4 + 4s^3 + (3+K_1)s^2 + 4s + (4+K_2)}$$

s^4	1	$3+K_1$	$4+K_2$
s^3	$\cancel{4}$	$\cancel{1}$	

$$\begin{array}{c|cc|c}
s^2 & | & 1 & 3+K_1 | & -2+K_1 & 4+K_2 \\ \hline & | & 1 & | & - & \\ & | & - & | & - & \\ \hline & | & 2+K_1 & 4+K_2 | & (4+K_2)-(2+K_1) & -2-K_2+K_1 \\ \hline s^1 & | & - & | & - & \\ \hline s^0 & 4+K_2 & & & &
\end{array}$$

so $2+K_1 > 0$ $K_1 > -2$

$4+K_2 > 0$ $K_2 > -4$

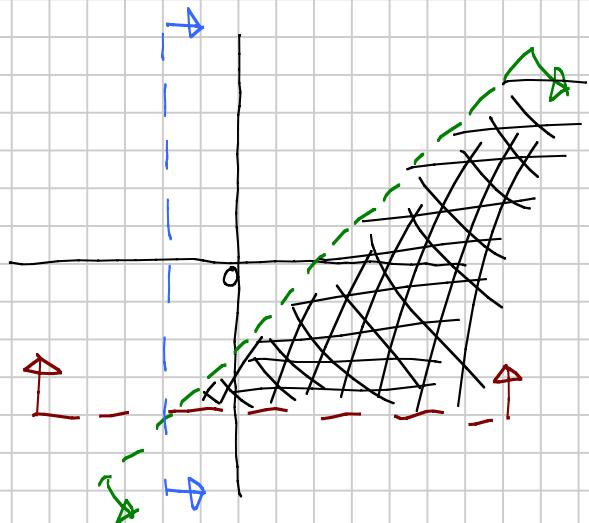
If $2+K_1 > 0, -2-K_2+K_1 > 0$
 $K_2 < K_1 - 2$

So limits defined by

$K_2 > -4, K_2 < K_1 - 2$

(c) $K_1 = 0$ gives pair of 0's in s' when $K_2 = -2$
 s^2 now is thus
 $2s^2 + (4-2) = 0$
 $s = \pm j$
 $\omega = 1 \text{ rad/s}$

(d) $K_2 = 0$ gives pair of 0's in s' when $K_1 = 2$
 s^2 now is thus
 $4s^2 + 4 = 0$
 $s = \pm j$
 $\omega = 1 \text{ rad/s}$



Problem 5

$$(a) (D) \bar{J}_M = \bar{J}_a + \bar{J}_1 + (\gamma_{21})^2 \bar{J}_2 \\ = 1 + 1 + (10)^2 200 = 4$$

$$D_M = D_a = 3$$

$$\frac{(\text{H})_m}{Ea} = \frac{\frac{1}{Im} \frac{Kt}{R_a}}{S(S + \frac{1}{Im}(D_m + \frac{Kt(K_b)}{R_a}))} = \frac{(4)(8)}{S(S + \frac{1}{4}(3) + (8)(\frac{1}{8}))} = \underline{\underline{\frac{2}{S(S+1)}}}$$

$$(2) \quad \frac{(\text{H})_2}{(\text{H})_1} = \frac{N_1}{N_2} = \frac{1}{10}$$

$$(3) \quad \frac{V_p}{(\frac{1}{4})_c} = \frac{80\pi}{(2 \text{ turns})(2\pi/\text{turn})} = 20$$

$$(b) T = \frac{G}{1+GH} = \frac{G_1 G_2}{1 + G_1 G_2 H K} = \left(\frac{\left(\frac{2}{(S)(S+1)}\right) \left(\frac{1}{10}\right)}{1 + \left(\frac{2}{(S)(S+1)}\right) \left(\frac{1}{10}\right) \left(\frac{20}{(K)}\right)} \right) \Leftrightarrow \frac{S(S+1)}{S(S+1)}$$

$$T = \frac{0.2}{s^2 + s + 4K}$$

$$(c) \begin{array}{ccc|c} & s & | & 4K \\ \bar{s} & | & & \\ s^0 & | & 4K & \end{array} \quad \text{so } \underline{\underline{4K > 0}}$$

(d) Step response: $w_n = \sqrt{4K} = 2$

$$2f_{wn} = 1, f = \frac{1}{2} = 0.25$$

$$T_5 = \frac{4}{\zeta^m} = \frac{4}{(\frac{4}{3})(2)} = \underline{8} \quad Tr_n = \text{[written value]} \quad (\text{arg for } \beta = -2^\circ, \gamma = 3^\circ)$$

$$T_r = \frac{T_{rn}}{C_{vn}} = \frac{1.262}{2} = 0.631$$

$$\% OS = e^{-\frac{P_{T1}}{\sqrt{1-P^2}}} \times 100 = 44.4\%$$

$$(e) \text{ 10% for OS means } \xi = \frac{-\ln(OS/100)}{\sqrt{\pi^2 + \ln^2(OS/100)}} = 0.591$$

$$2\sum w_n = 1, \text{ so } w_n = \frac{1}{(2)(1.591)} = .846$$

$$4 K = \omega_n^2, \quad K = \frac{\omega_n^2}{4} = 0,1789$$