## Duke University Edmund T. Pratt, Jr. School of Engineering

ECE 110L Spring 2018 Test II Michael R. Gustafson II

Name and NetID (please print)

In keeping with the Community Standard, I have neither provided nor received any assistance on this test. I understand if it is later determined that I gave or received assistance, I will be brought before the Undergraduate Conduct Board and, if found responsible for academic dishonesty or academic contempt, fail the class. I also understand that I am not allowed to speak to anyone except the instructor about any aspect of this test until the instructor announces it is allowed. I understand if it is later determined that I did speak to another person about the test before the instructor said it was allowed, I will be brought before the Undergraduate Conduct Board and, if found responsible for academic dishonesty or academic contempt, fail the class.

Signature:

### Instructions

First - please turn off any cell phones or other annoyance-producing devices. Vibrate mode is not enough - your device needs to be in a mode where it will make no sounds during the course of the test, including the vibrate buzz or those acknowledging receipt of a text or voicemail.

Please be sure to put each problem on its own page or pages - do *not* write answers to more than one problem on any piece of paper and do not use the back of a problem for work on a *different* problem. You will be turning in each of the problems independently. This cover page should be stapled to the front of Problem 1.

Make sure that your name *and* NetID are *clearly* written at the top of *every* page, just in case problem parts come loose in the shuffle. Make sure that the work you are submitting for an answer is clearly marked as such. Finally, when turning in the test, individually staple all the work for each problem and place each problem's work in the appropriate folder.

Note that there may be people taking the test after you, so you are not allowed to talk about the test - even to people outside of this class - until I send along the OK. This includes talking about the specific problem types, how long it took you, how hard you thought it was - really anything. Please maintain the integrity of this test.

You may use the || symbol for resistances in parallel and do not need to expand that construction. Be clear with your use of parentheses, however; simply writing something like

$$R_{\rm eq} = R_1 + R_2 \parallel R_3 + R_4$$

is too vague since it could refer to any of the four combinations below:



#### Problem I: [20 pts.] The (mostly) Basics

(1) Write the equations that relate the current to the voltage for the following three elements:



(2) For the reactive elements above, write the equation for the energy stored in each element:

(3) Circle the appropriate entries in the sentence below:

The (current through / voltage drop across) a capacitor and the (current through / voltage drop across) an inductor must be continuous.

(4) Clearly using phasors, simplify the following signal into a single cosine:

 $v_{\rm a}(t) = 3\sin(17t + 18^{\circ}) + 2\cos(17t - 41^{\circ})$ 

(5) Fill in the following table:

Quantity	Symbol	Name of real part	Symbol	Name of imaginary part	Symbol
Impedance					
Admittance					

(6) (This one is not as basic...) A resistor is connected to a capacitor inside a box, but you do not know if they are connected in series or parallel. Two terminals are sticking out of the box. You conduct two experiments by applying a sinusoidal voltage across the terminals and then measuring the steady state current going into the box. The inputs and outputs you measured are below:

 Source Voltage (V)
 Source Current (mA)

  $10 \cos(10t)$   $3.84 \cos(10t + 54.8^{o})$ 
 $10 \cos(20t)$   $5.44 \cos(20t + 35.4^{o})$ 

How are the resistor and capacitor connected? What are their values? *Hint:* look at the formulas for and values of the impedance and admittance.

# Problem II: [25 pts.] Switched Circuit

Given the following circuit:



where the switch has been *closed* for a very long time before t = 0 ms. The switch opens at 0 ms. Assuming

 $i_{\rm a} = 15 \text{ mA}$   $v_{\rm b} = 20 \text{ V}$   $R_1 = 2 \text{ k}\Omega$   $R_2 = 4 \text{ k}\Omega$  L = 10 mH  $C = 2\mu\text{F}$ 

- (1) At time  $t = 0^+$ , what is the current through the inductor?
- (2) At time  $t = 0^+$ , what is the voltage drop across the inductor? Use the passive sign convention relative to  $i_{\rm L}$  to label this voltage.
- (3) At time  $t = 0^+$ , what is the voltage drop across the capacitor?
- (4) At time  $t = 0^+$ , what is the current through the capacitor? Use the passive sign convention relative to  $v_{\rm C}$  to label this current.
- (5) Determine and accurately sketch an expression for the current through the inductor,  $i_{\rm L}$ , for t > 0 ms. You domain should be at least three time constants past the switch time. On the sketch, be sure to clearly indicate time constants and how you approximated slopes at various locations.
- (6) Determine and accurately sketch an expression for the voltage across the capacitor,  $v_{\rm C}$ , for t > 0 ms. You domain should be at least three time constants past the switch time. On the sketch, be sure to clearly indicate time constants and how you approximated slopes at various locations.

## Problem III: [20 pts.] Frequency and Steady-State Analysis

Given the following circuit:



- (1) Determine a transfer function  $\mathbb{H}_{a}(j\omega)$  between source  $\mathbb{V}_{a}$  and output  $\mathbb{I}_{x}$ . You should assume  $i_{b}$  is 0 A for this.
- (2) Determine a transfer function  $\mathbb{H}_{\mathbf{b}}(j\omega)$  between source  $\mathbb{I}_{\mathbf{b}}$  and output  $\mathbb{I}_{\mathbf{x}}$ . You should assume  $v_{\mathbf{a}}$  is 0 V for this.
- (3) What are the units of  $\mathbb{H}_{a}(j\omega)$ ? What are the units of  $\mathbb{H}_{b}(j\omega)$ ?
- (4) Assuming

$v_{\rm a}(t) = 6 + 4\cos(500t) + 8\cos(100000t)$ V	$R_1 = 4 \ \mathrm{k}\Omega$
$i_{\rm b}(t) = 5 + 2\cos(500t) {\rm mA}$	$R_2 = 1 \ \mathrm{k}\Omega$
	L = 50  mH

and further assuming the circuit has been place for a very long time prior to  $t = t_0$  sec, determine an expression in the time domain for the steady state voltage  $i_x(t)$  for  $t > t_0$  sec.

## Problem IV: [20 pts.] Thévenin-Norton Equivalents I

Given the following circuit:



- (1) Clearly determine the values of and draw the Thévenin equivalent for the circuit as seen from terminals  $\alpha$  and  $\beta$ . Note that you must explicitly solve for any variables you cannot leave a coupled system of expressions. Once you have fully solved for an unknown, however, you may use it later without needing to write in the substitution. Also, please do not spend time cleaning up expressions. Make sure  $\alpha$  and  $\beta$  are clearly drawn on your equivalent circuit.
- (2) Clearly determine the values of and draw the Norton equivalent for the circuit as seen from terminals  $\alpha$  and  $\beta$ . Same rules as above.
- (3) If a load resistor  $R_{\rm L}$  were placed between the terminals  $\alpha$  and  $\beta$ , what value of this resistor would maximize the power transferred to that load? What is the value of the power transferred to that load?

## Problem V: [15 pts.] Thévenin-Norton Equivalents II

Given the following circuit:



Clearly determine the values of and draw the Thévenin equivalent for the circuit as seen from terminals  $\xi$  and  $\eta$ . Note that you must explicitly solve for any variables - you cannot leave a coupled system of expressions. Once you have fully solved for an unknown, however, you may use it later without needing to write in the substitution. Also, please do not spend time cleaning up expressions. Make sure  $\xi$  and  $\eta$  are clearly drawn on your equivalent circuit.