

Duke University  
Edmund T. Pratt, Jr. School of Engineering

ECE 110 Spring 2013  
Test II  
Michael R. Gustafson II

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Name (please print) \_\_\_\_\_

In keeping with the Community Standard, I have neither provided nor received any assistance on this test. I understand if it is later determined that I gave or received assistance, I will be brought before the Undergraduate Conduct Board and, if found responsible for academic dishonesty or academic contempt, fail the class. I also understand that I am not allowed to speak to anyone except the instructor about any aspect of this test until the instructor announces it is allowed. I understand if it is later determined that I did speak to another person about the test before the instructor said it was allowed, I will be brought before the Undergraduate Conduct Board and, if found responsible for academic dishonesty or academic contempt, fail the class.

Signature: \_\_\_\_\_

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## Instructions

First - please turn **off** any cell phones or other annoyance-producing devices. Vibrate mode is not enough - your device needs to be in a mode where it will make no sounds during the course of the test, including the vibrate buzz or those acknowledging receipt of a text or voicemail.

Please be sure to put each problem on its own page or pages - do *not* write answers to more than one problem on any piece of paper and do not use the back of a problem for work on a *different* problem. You will be turning in each of the problems independently. This cover page should be stapled to the front of Problem 1.

Make sure that your name *and* NET ID are *clearly* written at the top of *every* page, just in case problem parts come loose in the shuffle. Make sure that the work you are submitting for an answer is clearly marked as such. Finally, when turning in the test, individually staple all the work for each problem and place each problem's work in the appropriate folder.

Note that there may be people taking the test after you, so you are not allowed to talk about the test - even to people outside of this class - until I send along the OK. This includes talking about the specific problem types, how long it took you, how hard you thought it was - really anything. Please maintain the integrity of this test.

Name (please print):

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**Problem I: [15 pts.] The Basics**

- (1) Clearly using phasors, simplify the following signal into a single cosine:

$$v_a(t) = 4 \cos(200t) + 7 \sin(200t)$$

- (2) Clearly using phasors, simplify the following signal into a single cosine:

$$i_b(t) = 9 \cos(1000t - 14^\circ) - 14 \sin(1000t + 22^\circ)$$

- (3) Assuming the network below is connected to a system where all sources are oscillating at a single angular frequency  $\omega=5000$  rad/s, find the admittance, conductance, impedance, reactance, resistance, and susceptance between terminals  $a$  and  $b$  if

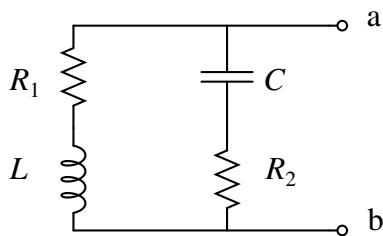
$$R_1 = 1000 \, \Omega$$

$$R_2 = 3000 \, \Omega$$

$$L = 0.4 \, \text{H}$$

$$C = 0.2 \, \mu\text{F}$$

Also, clearly indicate the most common letter used to represent each of the six quantities. For instance, you might write “Resistance:  $R$ ” (side note - you’re welcome).

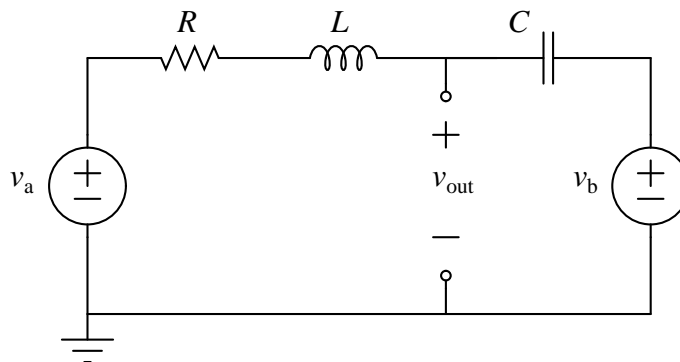


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**Problem II: [25 pts.] Frequency and Steady-State Analysis**

Given the following circuit:



- (1) Determine a transfer function  $\mathbb{H}_a(j\omega)$  between source  $v_a$  and output  $v_{out}$ . You should assume  $v_b$  is 0 for this.
- (2) Determine a transfer function  $\mathbb{H}_b(j\omega)$  between source  $v_b$  and output  $v_{out}$ . You should assume  $v_a$  is 0 for this.
- (3) What is the resonant frequency of this circuit?
- (4) Assuming

$$v_a(t) = 5 + 8 \cos(10000t + 12^\circ) \text{ V}$$

$$v_b(t) = 6 + 7 \sin(20000t - 28^\circ) \text{ V}$$

$$R = 3 \text{ k}\Omega$$

$$L = 200 \text{ mH}$$

$$C = 20 \text{ nF} = 20 \times 10^{-9} \text{ F}$$

and further assuming the circuit has been place for a very long time prior to  $t = t_0$  sec, determine an expression in the time domain for the voltage  $v_{out}(t)$  for  $t > t_0$  sec.

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**Problem III: [25 pts.] Fourier series analysis**

The Fourier series approximation for a particular signal with a period of 100 ms can be written as:

$$v_i(t) = 5 + \sum_{n=1}^{\infty} \frac{10}{(n+1)} \cos\left(20\pi nt + \frac{180^\circ}{n^2}\right)$$

- (1) Assume a signal with voltage  $v_i(t)$  is the input to a filter having a transfer function of:

$$\mathbb{H}(j\omega) = \frac{100}{j\omega + 150}$$

- (a) Determine the magnitudes and phases of the DC component and the first three harmonics of the output wave from this filter.
  - (b) Determine an expression in the time domain for the output wave from the filter that takes into account the DC component and the first three harmonics. Call this output wave  $v_o(t)$ .
  - (c) What kind of filter does  $\mathbb{H}(j\omega)$  represent? Why do you think that? What is the maximum gain of this filter? What is the half-power (cutoff) frequency of this filter?
- (2) Assume a signal with voltage  $v_i(t)$  is the input to an *ideal* low-pass filter with a pass-band gain and cutoff frequency equal to those of the filter described in  $\mathbb{H}(j\omega)$ . Determine an expression for the output wave from the ideal filter and call it  $v_x(t)$ .

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**Problem IV: [20 pts.] Bode plot approximations**

- (1) You are given a circuit to analyze and have properly come up with a transfer function for it of:

$$G(j\omega) = \frac{Y(j\omega)}{X(j\omega)} = \frac{100(j\omega)^2}{(j\omega + 200)(j\omega + 20000)}$$

- (a) Sketch a straight-line approximation for the magnitude portion of the Bode plot. Be sure to label the axes, including numerical values, along with all slopes and critical frequencies.
- (b) Sketch a straight-line approximation for the phase portion of the Bode plot. Be sure to label the axes, including numerical values, along with all slopes and critical frequencies.
- (c) Based on this and any other information at your disposal, what kind of filter do you believe this to be? Why do you believe that? You must provide some reasonable explanation in order to receive credit for this part.
- (d) *Approximately* what is/are the cutoff frequency/ies for this filter? How did you come to that conclusion? Describe the process you used or reference the Bode diagram.
- (e) Given the transfer function, find a differential equation that relates  $y(t)$  and its derivatives to  $x(t)$  and its derivatives.

- (2) You are given a circuit to analyze and have properly come up with a transfer function for it of:

$$H(j\omega) = \frac{Y(j\omega)}{X(j\omega)} = \frac{1000j\omega(j\omega + 10000)}{(j\omega + 10)(j\omega + 1000)(j\omega + 100000)}$$

- (a) Sketch a straight-line approximation for the magnitude portion of the Bode plot. Be sure to label the axes, including numerical values, along with all slopes and critical frequencies.
- (b) Based on this and any other information at your disposal, what kind of filter do you believe this to be? Why do you believe that? You must provide some reasonable explanation in order to receive credit for this part.
- (c) *Approximately* what is/are the cutoff frequency/ies for this filter? How did you come to that conclusion? Describe the process you used or reference the Bode diagram.

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**Problem V: [15 pts.] Filters**

- (1) Determine the transfer function for a first-order high-pass filter with a maximum gain of 1 and a cutoff frequency of 5000 Hz. Then, using only 100 nF capacitors and any reasonably sized resistors you need, design a voltage-to-voltage circuit that has this transfer function.
- (2) Determine the transfer function for a first-order low-pass filter with a maximum gain of 1 and a cutoff frequency of 1000 rad/s. Then, using only 100 nF capacitors and any reasonably sized resistors you need, design a voltage-to-voltage circuit that has this transfer function.