

Duke University  
Edmund T. Pratt, Jr. School of Engineering

ECE 110 Fall 2016  
**Test II**  
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Name (please print)\_\_\_\_\_

In keeping with the Community Standard, I have neither provided nor received any assistance on this test. I understand if it is later determined that I gave or received assistance, I will be brought before the Undergraduate Conduct Board and, if found responsible for academic dishonesty or academic contempt, fail the class. I also understand that I am not allowed to speak to anyone except the instructor about any aspect of this test until the instructor announces it is allowed. I understand if it is later determined that I did speak to another person about the test before the instructor said it was allowed, I will be brought before the Undergraduate Conduct Board and, if found responsible for academic dishonesty or academic contempt, fail the class.

Signature:\_\_\_\_\_

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## Instructions

First - please turn **off** any cell phones or other annoyance-producing devices. Vibrate mode is not enough - your device needs to be in a mode where it will make no sounds during the course of the test, including the vibrate buzz or those acknowledging receipt of a text or voicemail.

Please be sure to put each problem on its own page or pages - do *not* write answers to more than one problem on any piece of paper and do not use the back of a problem for work on a *different* problem. You will be turning in each of the problems independently. This cover page should be stapled to the front of Problem 1.

Make sure that your name *and* NET ID are *clearly* written at the top of *every* page, just in case problem parts come loose in the shuffle. Make sure that the work you are submitting for an answer is clearly marked as such. Finally, when turning in the test, individually staple all the work for each problem and place each problem's work in the appropriate folder.

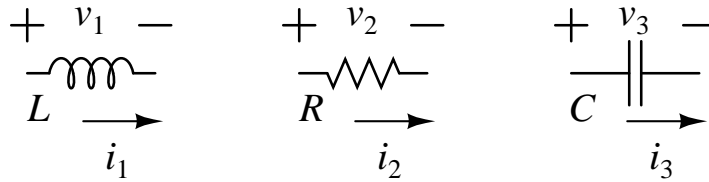
Note that there may be people taking the test after you, so you are not allowed to talk about the test - even to people outside of this class - until I send along the OK. This includes talking about the specific problem types, how long it took you, how hard you thought it was - really anything. Please maintain the integrity of this test.

Name (please print):

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### Problem I: [20 pts.] The (mostly) Basics

- (1) Write the equations that relate the current to the voltage for the following three elements:



- (2) For the reactive elements above, write the equation for the energy stored in each element:

- (3) Circle the appropriate entries in the sentence below:

The (voltage drop across / current through) an inductor and the (voltage drop across / current through) a capacitor must be continuous.

- (4) Clearly using phasors, simplify the following signal into a single cosine:

$$v_a(t) = 5 \cos(10t - 14^\circ) + 6 \sin(10t + 22^\circ)$$

- (5) Fill in the following table:

Quantity	Symbol	Name of real part	Symbol	Name of imaginary part	Symbol
Impedance					
Admittance					

- (6) (This one is not as basic...) A resistor is connected to an inductor inside a box, but you do not know if they are connected in series or parallel. Two terminals are sticking out of the box. You conduct two experiments by applying a sinusoidal voltage across the terminals and then measuring the steady state current going into the box. The inputs and outputs you measured are below:

Input Voltage (V)	Output Current (mA)
$5 \cos(5000t)$	$3.11 \cos(5000t - 43.1^\circ)$
$5 \cos(20000t)$	$2.33 \cos(20000t - 13.1^\circ)$

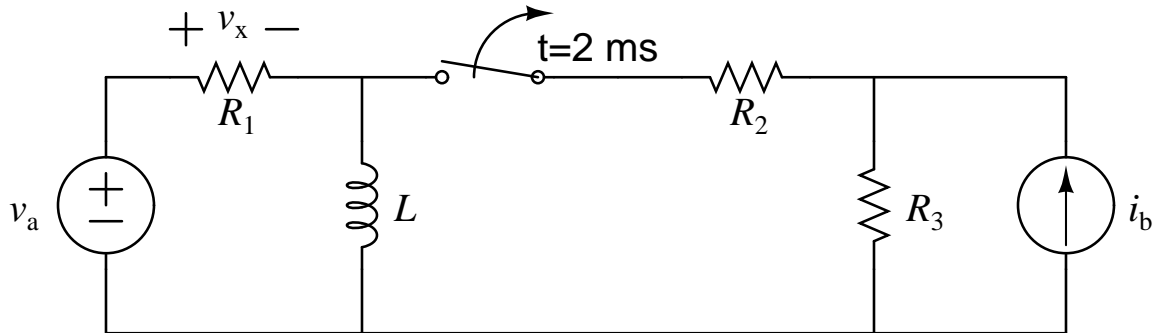
How are the resistor and inductor connected? What are their values? *Hint:* look at the formulas for and values of the impedance and admittance.

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## Problem II: [20 pts.] Switched Circuit

Given the following circuit:



where the switch has been *closed* for a very long time before  $t = 2$  ms. The switch opens at 2 ms. Assuming

$$v_a = 8 \text{ V} \quad i_b = 30 \text{ mA} \quad R_1 = 500 \, \Omega \quad R_2 = 4 \text{ k}\Omega \quad R_3 = 1 \text{ k}\Omega \quad L = 750 \text{ mH}$$

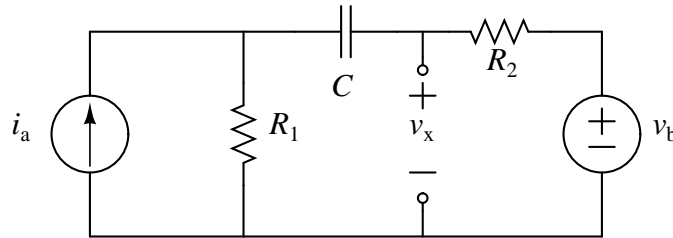
Determine and accurately sketch an expression for the voltage drop across resistor  $R_1$ ,  $v_x$ , for  $t > 0$  ms. Your domain should be at least three time constants past the switch time. On the sketch, be sure to clearly indicate time constants and how you approximated slopes at various locations. **Note:** if you get a constant value of  $v_x$  for all time, you are doing it wrong.

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### Problem III: [22 pts.] Frequency and Steady-State Analysis

Given the following circuit:



- (1) Determine a transfer function  $\mathbb{H}_a(j\omega)$  between source  $\mathbb{I}_a$  and output  $\mathbb{V}_x$ . You should assume  $v_b$  is 0 V for this.
- (2) Determine a transfer function  $\mathbb{H}_b(j\omega)$  between source  $\mathbb{V}_b$  and output  $\mathbb{V}_x$ . You should assume  $i_a$  is 0 A for this. (Spoiler Alert - you are going to use this in parts (4) and (5) below).
- (3) Assuming

$$i_a(t) = 4 \cos(2t) + 7 \cos(35t) \text{ mA}$$

$$v_b(t) = 6 + 4 \cos(35t) \text{ V}$$

$$R_1 = 2 \text{ k}\Omega$$

$$R_2 = 8 \text{ k}\Omega$$

$$C = 5 \text{ }\mu\text{F} = 5 \times 10^{-6} \text{ F}$$

and further assuming the circuit has been place for a very long time prior to  $t = t_0$  sec, determine an expression in the time domain for the steady state voltage  $v_x(t)$  for  $t > t_0$  sec.

**Note for the next two parts you are being asked about  $\mathbb{H}_b(j\omega)$**

**The transfer function related to the voltage source**

**The one related to the source on the right<sup>1</sup>**

- (4) Sketch a straight line approximation for the Bode magnitude plot of  $\mathbb{H}_b(j\omega)$ . Be sure to label corners, slopes, and magnitudes.
- (5) Given the information above, what kind of filter does  $\mathbb{H}_b(j\omega)$  represent? Why do you think that?

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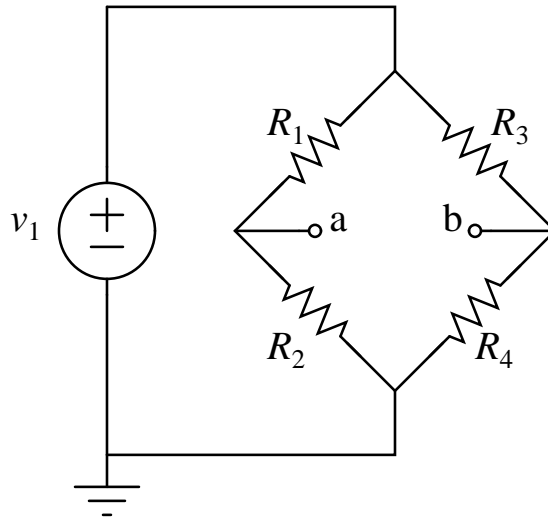
<sup>1</sup>In other words, not the “a” one.

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### Problem IV: [14 pts.] Thévenin-Norton Equivalents

Given the following circuit:



- (1) Clearly determine the values of and draw the Thévenin equivalent for the circuit as seen from terminals  $a$  and  $b$ . Note that you must explicitly solve for any variables - you cannot leave a coupled system of expressions. Once you have fully solved for an unknown, however, you may use it later without needing to write in the substitution. Also, please do not spend time cleaning up expressions. Make sure  $a$  and  $b$  are clearly drawn on your equivalent circuit.
- (2) Clearly determine the values of and draw the Norton equivalent for the circuit as seen from terminals  $a$  and  $b$ . Same rules as above.
- (3) If a load resistor  $R_L$  were placed between the terminals  $a$  and  $b$ , what value of this resistor would maximize the power transferred to that load? What is the value of the power transferred to that load?

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### Problem V: [24 pts.] Bode Plots and Filters

- (1) You are given a circuit to analyze and have properly come up with a transfer function for it of:

$$\mathbb{H}(j\omega) = \frac{\mathbb{Y}(j\omega)}{\mathbb{X}(j\omega)} = \frac{960000(j\omega)}{(j\omega + 40)(j\omega + 6000)} = \frac{4(j\omega)}{(1 + j\frac{\omega}{40})(1 + j\frac{\omega}{6000})}$$

- (a) Sketch a straight-line approximation for the magnitude portion of the Bode plot. Be sure to label the axes, including numerical values, along with all slopes and critical frequencies.
- (b) Sketch a straight-line approximation for the phase portion of the Bode plot. Be sure to label the axes, including numerical values, along with all slopes and critical frequencies.
- (c) Based on this and any other information at your disposal, what kind of filter do you believe this to be? Why do you believe that? You must provide some reasonable explanation in order to receive credit for this part.
- (d) *Approximately* what is/are the cutoff frequency/ies for this filter? How did you come to that conclusion? Describe the process you used or reference the Bode diagram.
- (e) What is the passband gain for this filter?
- (2) Design a voltage-to-voltage low pass filter with a maximum gain of 1 and a cutoff frequency of 4000 rad/s using a single 2.2 k $\Omega$  resistor, a single reactive element of your choice, and wires.
- (3) Design a voltage-to-voltage high pass filter with a maximum gain of 1 and a cutoff frequency of 4000 rad/s using a single 2.2 k $\Omega$  resistor, a single reactive element of the kind you did **not** use in (2), and wires. Which is to say, if you used a capacitor in (2) you must use an inductor here; if an inductor there then a capacitor here.
- (4) Given the straight-line approximation to the Bode magnitude plot below, and also given that any double corners below are represented by critically damped (i.e. repeated) roots, determine a reasonable expression for the transfer function  $\mathbb{G}(j\omega)$  from which it was generated. *Carefully* note where the corners are. Also state what kind of filter you believe this is, why you believe that, *approximately* what the cutoff frequency/frequencies is/are, and what the passband gain is/is.

