

Duke University  
Edmund T. Pratt, Jr. School of Engineering

ECE 110 Fall 2015  
Test II  
Michael R. Gustafson II

---

Name (please print) \_\_\_\_\_

In keeping with the Community Standard, I have neither provided nor received any assistance on this test. I understand if it is later determined that I gave or received assistance, I will be brought before the Undergraduate Conduct Board and, if found responsible for academic dishonesty or academic contempt, fail the class. I also understand that I am not allowed to speak to anyone except the instructor about any aspect of this test until the instructor announces it is allowed. I understand if it is later determined that I did speak to another person about the test before the instructor said it was allowed, I will be brought before the Undergraduate Conduct Board and, if found responsible for academic dishonesty or academic contempt, fail the class.

Signature: \_\_\_\_\_

---

## Instructions

First - please turn **off** any cell phones or other annoyance-producing devices. Vibrate mode is not enough - your device needs to be in a mode where it will make no sounds during the course of the test, including the vibrate buzz or those acknowledging receipt of a text or voicemail.

Please be sure to put each problem on its own page or pages - do *not* write answers to more than one problem on any piece of paper and do not use the back of a problem for work on a *different* problem. You will be turning in each of the problems independently. This cover page should be stapled to the front of Problem 1.

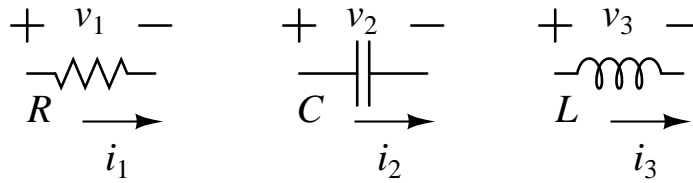
Make sure that your name *and* NET ID are *clearly* written at the top of *every* page, just in case problem parts come loose in the shuffle. Make sure that the work you are submitting for an answer is clearly marked as such. Finally, when turning in the test, individually staple all the work for each problem and place each problem's work in the appropriate folder.

Note that there may be people taking the test after you, so you are not allowed to talk about the test - even to people outside of this class - until I send along the OK. This includes talking about the specific problem types, how long it took you, how hard you thought it was - really anything. Please maintain the integrity of this test.

Name (please print):  
Community Standard (print ACPUB ID):

**Problem I: [16 pts.] The Basics**

- (1) Write the equations that relate the current to the voltage for the following three elements:



- (2) For the reactive elements above, write the equation for the energy stored in each element:

- (3) Circle the appropriate entries in the sentence below:

The (voltage drop across / current through) a capacitor and the  
(voltage drop across / current through) an inductor must be continuous.

- (4) Clearly using phasors, simplify the following signal into a single cosine:

$$v_a(t) = 6 \cos(25t) - 4 \sin(25t)$$

- (5) Clearly using phasors, simplify the following signal into a single cosine:

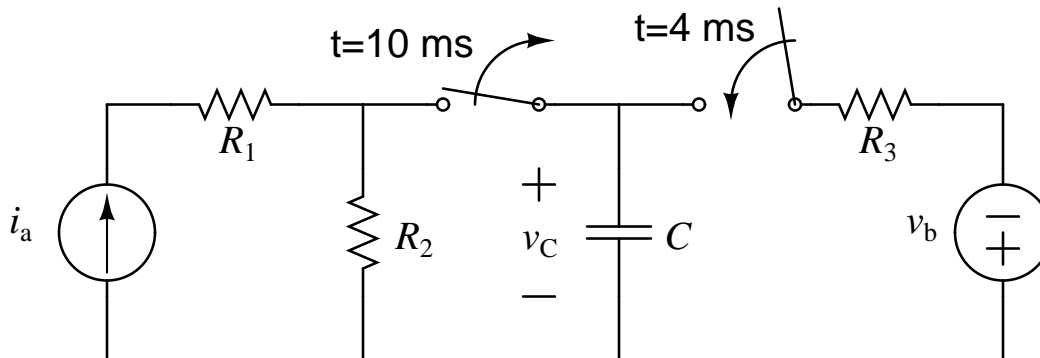
$$i_b(t) = 17 \sin(200t + 41^\circ) + 10 \cos(200t - 15^\circ)$$

- (6) A resistor  $R$  is in parallel with a reactive element. At a frequency of 500 rad/s, the impedance of the parallel combination is found to be  $200 + j400 \, \Omega$ . Draw the parallel combination below, including the actual values of the resistor and the reactive element.

Name (please print):  
Community Standard (print ACPUB ID):

**Problem II: [24 pts.] Switched Circuit**

Given the following circuit:



where the left switch has been *closed* for a very long time before  $t = 10$  ms and the right switch has been *open* for a very long time before  $t = 4$  ms. The right switch closes at 4 ms and the left switch opens at 10 ms. Assuming

$$i_a = 3 \text{ mA} \quad v_b = 10 \text{ V} \quad R_1 = 2 \text{ k}\Omega \quad R_2 = 2 \text{ k}\Omega \quad R_3 = 2 \text{ k}\Omega \quad C = 3 \text{ }\mu\text{F}$$

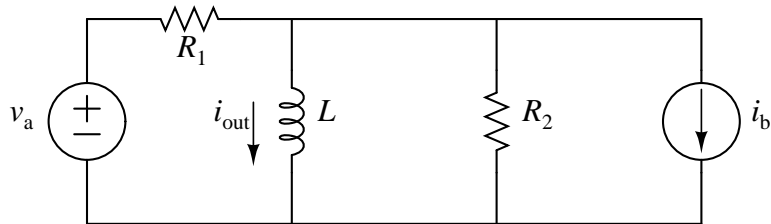
Determine and accurately sketch expressions for the capacitor voltage for all times  $0 \leq t \leq 30$  ms. On the sketch, be sure to clearly indicate time constants and how you approximated slopes at various locations.

Name (please print):

Community Standard (print ACPUB ID):

### Problem III: [24 pts.] Frequency and Steady-State Analysis

Given the following circuit:



- (1) Determine a transfer function  $\mathbb{H}_a(j\omega)$  between source  $v_a$  and output  $i_{out}$ . You should assume  $i_b$  is 0 A for this.
- (2) Determine a transfer function  $\mathbb{H}_b(j\omega)$  between source  $i_b$  and output  $i_{out}$ . You should assume  $v_a$  is 0 V for this.
- (3) Assuming

$$v_a(t) = 5 + 10 \cos(\omega_1 t) \text{ V}$$

$$i_b(t) = 7 + 8 \cos(\omega_2 t) \text{ mA}$$

$$R_1 = 1 \text{ k}\Omega$$

$$R_2 = 2 \text{ k}\Omega$$

$$L = 5 \text{ mH} = 5 \times 10^{-3} \text{ mH}$$

and further assuming the circuit has been placed for a very long time prior to  $t = t_0$  sec, determine an expression in the time domain for the current  $i_{out}(t)$  for  $t > t_0$  sec if:

(a)  $\omega_1 = \omega_2 = 100000 \text{ rad/s} = 1 \times 10^5 \text{ rad/s}$ .

(b)  $\omega_1 = 100000 \text{ rad/s} = 1 \times 10^5 \text{ rad/s}$  and  $\omega_2 = 200000 \text{ rad/s} = 2 \times 10^5 \text{ rad/s}$ .

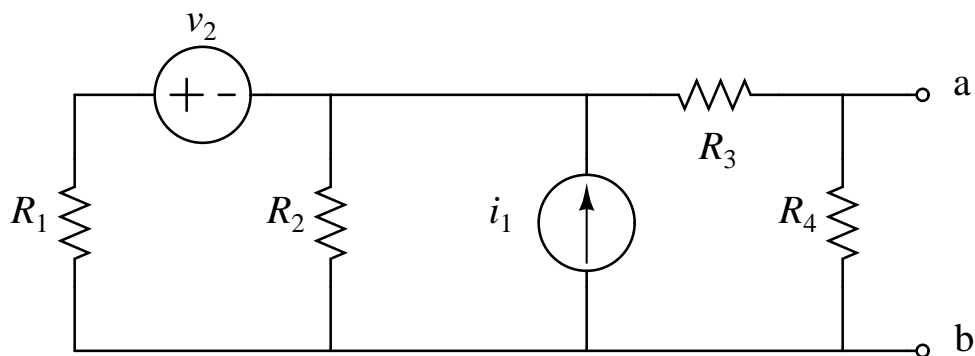
- (4) Given the element values above, what kind of filter does  $\mathbb{H}_a(j\omega)$  represent? Why do you think that?
- (5) Sketch a straight line approximation for the Bode magnitude plot of  $\mathbb{H}_a(j\omega)$ . Be sure to label corners, slopes, and magnitudes.

Name (please print):

Community Standard (print ACPUB ID):

**Problem IV: [16 pts.] Thévenin-Norton Equivalents**

Given the following circuit:



- (1) Clearly determine the values of and draw the Thévenin equivalent for the circuit as seen from terminals a and b.
- (2) Clearly determine the values of and draw the Norton equivalent for the circuit as seen from terminals a and b.
- (3) If a load resistor  $R_L$  were placed between the terminals a and b, what value of this resistor would maximize the power transferred to that load? What is the value of the power transferred to that load?

### Problem V: [20 pts.] Bode Plots

- (1) You are given a circuit to analyze and have properly come up with a transfer function for it of:

$$\mathbb{H}(j\omega) = \frac{\mathbb{Y}(j\omega)}{\mathbb{X}(j\omega)} = 10 \frac{(j\omega + 10000)(j\omega + 100000)}{(j\omega + 100)(j\omega + 1000000)} = 10 \frac{(j\omega + 10^4)(j\omega + 10^5)}{(j\omega + 10^2)(j\omega + 10^6)}$$

- (a) Sketch a straight-line approximation for the magnitude portion of the Bode plot. Be sure to label the axes, including numerical values, along with all slopes and critical frequencies.
- (b) Based on this and any other information at your disposal, what kind of filter do you believe this to be? Why do you believe that? You must provide some reasonable explanation in order to receive credit for this part.
- (c) *Approximately* what is/are the cutoff frequency/ies for this filter? How did you come to that conclusion? Describe the process you used or reference the Bode diagram.
- (d) What is the passband gain for this filter?
- (e) Given the transfer function, find a differential equation that relates  $y(t)$  and its derivatives to  $x(t)$  and its derivatives.
- (2) Given the straight-line approximation to the Bode magnitude plot below, and also given that all corners below are represented by critically damped ( $\zeta=1$ ) systems, determine a reasonable expression for the transfer function  $\mathbb{G}(j\omega)$  from which it was generated. *Carefully* note where the corners are. Also state what kind of filter you believe this is, why you believe that, *approximately* what the cutoff frequency/frequencies is/are, and what the passband gain is/is.

