#### Duke Unibersity Edmund T. Pratt, Jr. School of Engineering

ECE 110 Fall 2012 Test II Michael R. Gustafson II

Name (please print)

In keeping with the Community Standard, I have neither provided nor received any assistance on this test. I understand if it is later determined that I gave or received assistance, I will be brought before the Undergraduate Conduct Board and, if found responsible for academic dishonesty or academic contempt, fail the class. I also understand that I am not allowed to speak to anyone except the instructor about any aspect of this test until the instructor announces it is allowed. I understand if it is later determined that I did speak to another person about the test before the instructor said it was allowed, I will be brought before the Undergraduate Conduct Board and, if found responsible for academic dishonesty or academic contempt, fail the class.

Signature:

### Instructions

First - please turn **off** any cell phones or other annoyance-producing devices. Vibrate mode is not enough - your device needs to be in a mode where it will make no sounds during the course of the test, including the vibrate buzz or those acknowledging receipt of a text or voicemail.

Please be sure to put each problem on its own page or pages - do *not* write answers to more than one problem on any piece of paper and do not use the back of a problem for work on a *different* problem. You will be turning in each of the problems independently. This cover page should be stapled to the front of Problem 1.

Make sure that your name *and* NET ID are *clearly* written at the top of *every* page, just in case problem parts come loose in the shuffle. Make sure that the work you are submitting for an answer is clearly marked as such. Finally, when turning in the test, individually staple all the work for each problem and place each problem's work in the appropriate folder.

Note that there may be people taking the test after you, so you are not allowed to talk about the test - even to people outside of this class - until I send along the OK. This includes talking about the specific problem types, how long it took you, how hard you thought it was - really anything. Please maintain the integrity of this test.

You may use the || symbol for resistances in parallel and do not need to expand that construction. Be clear with your use of parentheses, however; simply writing something like

$$R_{\rm eq} = R_1 + R_2 \parallel R_3 + R_4$$

is too vague since it could refer to any of the four combinations below:



The title of each problem is an anagram of the focus of that problem.

# Problem I: [20 pts.] As in, instant relays<sup>1</sup>

This problem involves the following circuit



with a switch that is initially open for a very long time before t = 0 sec, closes at t = 0 sec, and remains closed from that point forward. Assuming that the voltage  $v_1(t)$  is a constant 10 V, the voltage  $v_2(t)$  is a constant -75 V, and the values of the reactive elements are

$$R_1 = 2000 \ \Omega = 2 \ \text{k}\Omega$$
  $R_2 = 5000 \ \Omega = 5 \ \text{k}\Omega$   $L = 0.010 \ \text{H} = 10 \ \text{mH}$ 

- (1) Determine mathematical expressions for the current through the inductor,  $i_{\rm L}(t)$ , for all times. Clearly show your work in doing so.
- (2) Make an accurate sketch of the current through the inductor for times between a time just before 0 sec and a time where the inductor current gets to be extremely close (within about 5% of its total change from time 0 to time  $\infty$ ) to its final value. Be sure to label your axes and show how you made an accurate sketch.

## Problem II: [20 pts.] As ant says, "Ace test daily!"<sup>2</sup>

Given the following circuit:

with



and assuming the circuit has been place for a very long time prior to  $t = t_0$  sec, determine an expression in the time domain for the current  $i_x(t)$  for  $t > t_0$  sec.

 $<sup>^2\</sup>mathrm{AC}$  steady state analysis

#### Problem III: [20 pts.] As I seriously sneer, "Fair!"<sup>3</sup>

The Fourier series approximation for a full-wave rectified sine wave with a period of 20 ms and an amplitude of 4 can be written as:

$$v_{\rm i}(t) = \frac{4}{\pi} + \sum_{n=1}^{\infty} \frac{8}{\pi (4n^2 - 1)} \cos(100\pi nt + 180^o)$$

(1) Assume a signal with voltage  $v_i(t)$  is the input to a filter having a transfer function of:

$$\mathbb{H}(j\omega) = \frac{j\omega}{j\omega + 500}$$

- (a) Determine the magnitudes and phases of the DC component and the first three harmonics of the output wave from this filter.
- (b) Determine an expression in the time domain for the output wave from the filter that takes into account the DC component and the first three harmonics. Call this output wave  $v_{\rm o}(t)$ .
- (c) What kind of filter does  $\mathbb{H}(j\omega)$  represent? Why do you think that?
- (2) Assume a signal with voltage  $v_i(t)$  is the input to an ideal low-pass filter with a pass-band gain of 1 and a cutoff frequency of 750 rad/s. Determine an expression for the output wave from the ideal filter and call it  $v_x(t)$ .

 $<sup>^3\</sup>mathrm{Fourier}$  series analysis

#### Problem IV: [20 pts.] Paradoxes bloom in pot pit<sup>4</sup>

You are given a circuit to analyze and have properly come up with a transfer function for it of:

$$\mathbb{H}(j\omega) = \frac{\mathbb{Y}(j\omega)}{\mathbb{X}(j\omega)} = \frac{20000(j\omega+10)}{(j\omega+1000)(j\omega+20000)}$$

- (1) Sketch a straight-line approximation for the magnitude portion of the Bode plot. Be sure to label the axes, including numerical values, along with all slopes and critical frequencies.
- (2) Based on this and any other information at your disposal, what kind of filter do you believe this to be? Why do you believe that? You must provide some reasonable explanation in order to receive credit for this part.
- (3) Approximately what is/are the cutoff frequency/ies for this filter? How did you come to that conclusion? Describe the process you used or reference the Bode diagram.
- (4) Given the transfer function, find a differential equation that relates y(t) and its derivatives to x(t) and its derivatives.

 $<sup>^4\</sup>mathrm{Bode}$  plot approximations

## Problem V: [20 pts.] Bats pilfer sand<sup>5</sup>

Determine the transfer function for a second-order band-pass filter with a logarithmic center frequency of 10000 rad/s, a quality of 0.2, and a pass-band gain of 1. For this filter:

- (1) Determine the damping ratio, linear center frequency, bandwidth, and cutoff frequencies for the filter. Be sure to clearly indicate which value is which and include units when appropriate.
- (2) Write the transfer function for the band-pass filter using one of the two "standard" forms we discussed in class for band-pass filters.
- (3) Assuming you have access to a single 100  $\mu$ F capacitor, along with a generous array of resistors and inductors design a voltage-to-voltage circuit that has the same transfer function as this filter. Be sure to clearly indicate where the output voltage is as well as what the component values are.