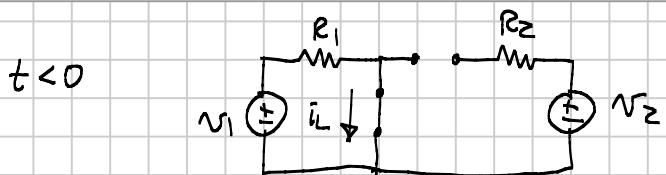
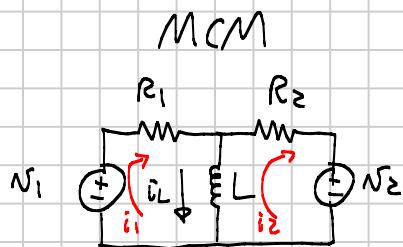
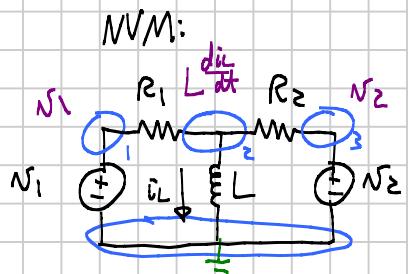
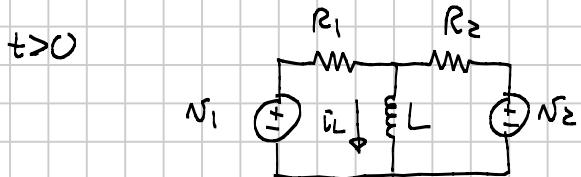


# ECE 110 Fall 2012 Test P

Note Title



$$i_L = \frac{V_1}{R_1} = 5\text{mA for } t > 0$$



$$\text{KCL at } V_2: \frac{d\bar{i}_2}{dt} - V_1 + \frac{d\bar{i}_2}{dt} - V_2 + i_L = 0$$

$$\text{KVL at } V_1: -V_1 + R_1 i_1 + L \frac{d(i_1 - i_2)}{dt} = 0$$

$$\left(\frac{L}{R_1} + \frac{L}{R_2}\right) \frac{d\bar{i}_2}{dt} + i_L = \frac{V_1}{R_1} + \frac{V_2}{R_2}$$

$$\text{KVL at } V_2: +L \frac{d(i_2 - i_1)}{dt} + R_2 i_2 + V_2 = 0$$

$$\text{AUX: } i_L = i_1 - i_2$$

$$\begin{aligned} -V_1 + R_1 i_1 + L \frac{d\bar{i}_2}{dt} &= 0 \\ -L \frac{d\bar{i}_2}{dt} + R_2(i_1 - i_L) + V_2 &= 0 \end{aligned}$$

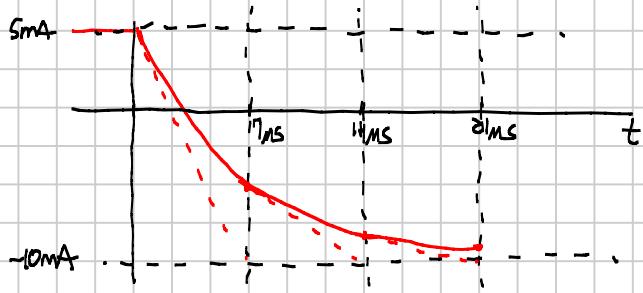
$$i_1 = \frac{V_1}{R_1} - \frac{L}{R_1} \frac{d\bar{i}_2}{dt}$$

$$-L \frac{d\bar{i}_2}{dt} + R_2 \left( \frac{V_1}{R_1} - \frac{L}{R_1} \frac{d\bar{i}_2}{dt} - i_L \right) + V_2 = 0$$

$$\bar{i}_L(t) = -10 + 15 e^{-t/\tau_{MS}} \text{ mA}$$

$$\bar{i}_L(t) = \begin{cases} t < 0 & 5\text{mA} \\ t > 0 & -10 + 15 e^{-t/\tau_{MS}} \text{ mA} \end{cases}$$

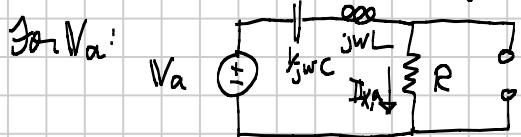
$$\left( \frac{L}{R_1} + \frac{L}{R_2} \right) \frac{d\bar{i}_2}{dt} + i_L = \frac{V_1}{R_1} + \frac{V_2}{R_2}$$



Problem 8

Can use superposition along with phasor analysis;

For  $V_a$ :



$$I_{x,a} = \frac{V_a}{jwL + R + \frac{1}{jwC}} = \frac{jwC V_a}{(jw)^2 LC + jwCR + 1}$$

$$V_a = 10 \quad w = 0 \quad V_a = 10 \angle 0^\circ$$

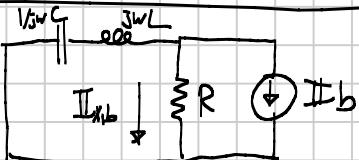
$$I_{x,a1} = \frac{j(0)C \cdot 10 \angle 0^\circ}{(j0)^2 LC + j(0)CR + 1} = 0 \quad i_{x,a1} = 0$$

$$V_a = 8 \cos 20t \angle 12^\circ \quad w = 20 \quad V_a = 8 \angle 12^\circ$$

$$I_{x,a2} = \frac{-j(20)(15 \cdot 10^{-6}) \cdot 8 \angle 12^\circ}{(j20)^2 (10 \cdot 10^{-3})(15 \cdot 10^{-6}) + (j20)(15 \cdot 10^{-6})(3 \cdot 10^3) + 1} = -1.78 \cdot 10^{-3} \angle 60^\circ$$

$$i_{x,a2} = -1.78 \cos(20t + 60^\circ)$$

$I_b$



$$I_{x,b} = \frac{-(jwL + \frac{1}{jwC}) I_b}{jwL + \frac{1}{jwC} + R} = \frac{-(jw)^2 LC + 1}{(jw)^2 LC + jwCR + 1} I_b$$

$$i_b = 12 \quad w = 0 \quad I_b = 12 \angle 0^\circ \text{ mA}$$

$$I_{x,b1} = \frac{-(0+1)}{j(0+0+1)} I_{b1} = -I_{b1} = -12 \angle 0^\circ \quad i_{x,b1} = -12 \text{ mA}$$

$$I_x = I_{x,a2} + i_{x,a2} + i_{x,b} = -1.78 \cos(20t + 60^\circ) - 12 \quad \underline{\text{mA}}$$

# Problem B

$$T = 0.02 \quad \omega_0 = \frac{2\pi}{0.02} = 100\pi \quad A = 4$$

$$a_0 = \frac{4}{\pi} \quad A_n = \frac{8}{\pi(4n^2-1)} \quad \phi_n = 180^\circ$$

| n | $\omega_0$ | Input  |             |                            | Output |  |
|---|------------|--------|-------------|----------------------------|--------|--|
|   |            | $A_n$  | $\phi_n$    | $H(j\omega_0)$             | $A_n$  | $\phi_n$                                 |
| 0 | 0          | 1.2732 | 0           | $0 \angle 0^\circ$         | 0      | 0  |
| 1 | $100\pi$   | 0.848  | $180^\circ$ | $0.532 \angle 57.86^\circ$ | 0.451  | $\angle 237.86^\circ$ or $-122.14^\circ$ |
| 2 | $200\pi$   | 0.170  | $180^\circ$ | $0.782 \angle 38.51^\circ$ | 0.133  | $\angle 218.51^\circ$ or $-141.49^\circ$ |
| 3 | $300\pi$   | 0.0726 | $180^\circ$ | $0.883 \angle 27.95^\circ$ | 0.0641 | $\angle 207.95^\circ$ or $-152.05^\circ$ |

(b)  $v_o(t) = 0.451 \cos(100\pi t - 122.14^\circ) + 0.133 \cos(200\pi t - 141.49^\circ) + 0.0641 \cos(300\pi t - 152.05^\circ) \dots V$

(c) High-pass;  $|H(j\omega)| \rightarrow 0$  and  $|H(j\omega)| \rightarrow 1$  ;  
 $|H|$  increases as  $w$  increases

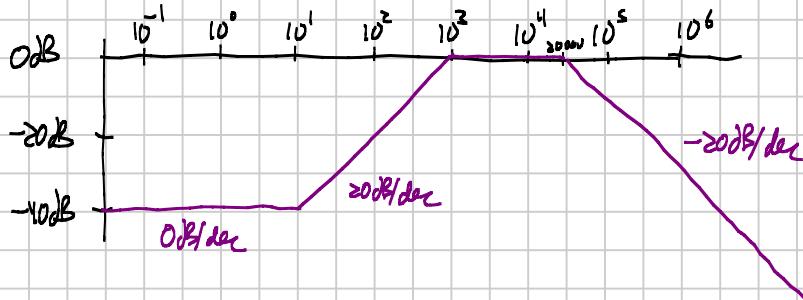
Bode of 

(d) Cutoff at 750 means only 0,  $100\pi$ ,  $200\pi$  get through;

$$v_x(t) = 1.2732 + 0.848 \cos(100\pi t + 180^\circ) + 0.170 \cos(200\pi t + 180^\circ) \text{ V}$$

## Problem IV

(1) Starts at lims  $|H(j\omega)| = \frac{20000 \cdot 10}{1000 \cdot 20000} = \frac{1}{100} \rightarrow -40 \text{dB}$



(2) Band-pass portion between  $-3 \text{dB}$  and  $0 \text{dB}$  is bounded on both sides

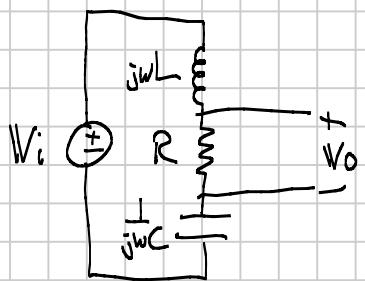
(3)  $\sim 1000 \text{ rad/s}$  and  $\sim 20000 \text{ rad/s}$  based on  $3 \text{dB}$  "corners"  
(actually, a little outside these values)

$$\frac{Y}{X} = \frac{2 \cdot 10^5 + 2 \cdot 10^4 (j\omega)}{2 \cdot 10^7 + (2 \cdot 1000) (j\omega) + (j\omega)^2}$$

$$(j\omega)^2 + 2 \cdot 1000 j\omega + 2 \cdot 10^7 Y = (2 \cdot 10^4 j\omega + 2 \cdot 10^5) X$$

$$\left. \frac{d^2}{dt^2} y + 2 \cdot 1000 \frac{dy}{dt} + 2 \cdot 10^7 y = 2 \cdot 10^4 \frac{dx}{dt} + 2 \cdot 10^5 X \right]$$

# Problem 5



$$H = \frac{R}{jwL + R + \frac{1}{jwC}} = \frac{jwCR}{Qw^2LC + jwCR + 1} = \frac{jw^2R/L}{Qw^2 + (jw)R/L + 1/LC}$$

denom:  $(jw)^2 + 2\zeta w_n jw + w_n^2$

$$w_n = 10^4 \text{ rad/s} \quad Q = 0.2 \quad \zeta = \frac{1}{2Q} = 0.5$$

(1) \*  $\zeta = 1/2Q = 0.5$

\*  $w_{\text{resonant}} = \frac{w_n \sqrt{1+4Q^2}}{2\zeta} = 26925.8 \text{ rad/s}$

\*  $BW = 2\zeta w_n = 50000 \text{ rad/s}$

\*  $w_{co} = w_{\text{resonant}} \pm BW/2 = 19258, 51925.8 \text{ rad/s}$

(2)

$$\frac{K^2 w_{\text{resonant}}}{(jw)^2 + 2\zeta w_n jw + w_n^2}$$

$$\frac{K}{1 + jQ(\frac{w}{w_n} - \frac{w_n}{w})}$$

$$\frac{5 \cdot 10^4 jw}{(jw)^2 + 5 \cdot 10^4 jw + 1 \cdot 10^8}$$

$$\frac{1}{1 + j(0.2)(\frac{w}{10^4} - \frac{10^4}{w})}$$

(3)  $C = 100 \mu F$

$$\frac{1}{LC} = w_n^2 \quad \Rightarrow \quad L = \frac{1}{C w_n^2} = 100 \mu H \quad \underline{\text{or } 0.1 mH}$$

$$R = 2\zeta w_n \quad \text{so} \quad R = 2\zeta w_n L = (5 \cdot 10^4)(1 \cdot 10^{-4}) = 5 \Omega$$