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Thursday, July 22, 2021

$$1) \quad S_r(s) = \frac{200}{(s)(s+2)(s+50)} \quad \lim_{t \rightarrow \infty} s_r(t) = \lim_{s \rightarrow 0} s S_r(s) = 2$$

DOMINANT POLE AT -2,  $T_s \approx \frac{4}{2} = 2 \text{ sec}$ ; POLE AT -50 FAR AWAY!

$$2) \quad T = \frac{G}{1+G} = \frac{\frac{5}{s+4}}{1 + \frac{5}{s+4}} = \frac{5}{s+9}$$

$$S_r(s) = \frac{5}{(s)(s+9)} \quad \lim_{t \rightarrow \infty} s_r(t) = \lim_{s \rightarrow 0} s S_r(s) = \frac{5}{9}$$

DOMINANT POLE AT -9,  $T_s \approx \frac{4}{9}$

$$3) \quad \omega_n = \sqrt{35} = 5.916 \quad 2\zeta\omega_n = 4 \quad \zeta = 0.3381$$

$$c_{step}(\infty) = \lim_{s \rightarrow 0} s \left( \frac{I}{s} \right) = \frac{25}{35} = 0.7143 \quad T_s = \frac{4}{\zeta\omega_n} = \frac{8}{2\zeta\omega_n} = \frac{8}{4} = 2 \quad \%OS = \exp\left(-\frac{\zeta\pi}{\sqrt{1-\zeta^2}}\right) \times 100\% = 32.35\%$$

$$\omega_n = 5.916 \text{ rad/s} \quad \zeta = 0.3381 \quad \omega_d = \omega_n \sqrt{1-\zeta^2} = 5.568 \text{ rad/s}$$

$$4) \quad T = \frac{G}{1+G} = \frac{35}{s^2 + s + 40} \quad \omega_n = \sqrt{40} = 6.325 \quad 2\zeta\omega_n = 1 \quad \zeta = \frac{1}{2\omega_n} = 0.07906$$

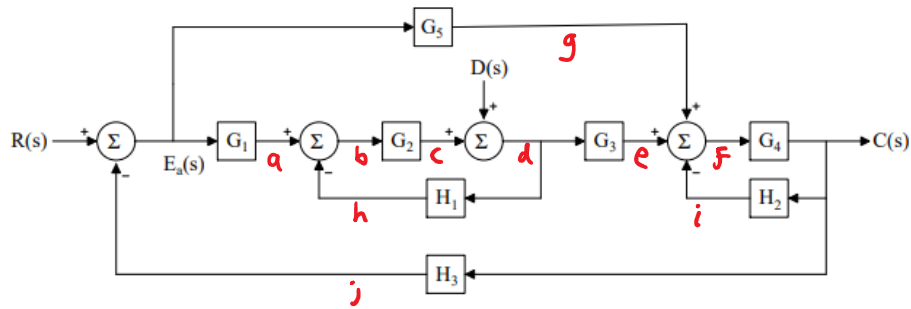
$$c_{step}(\infty) = \lim_{s \rightarrow 0} s \left( \frac{I}{s} \right) = \frac{35}{40} = 0.875 \quad T_s = \frac{8}{2\zeta\omega_n} = 8 \text{ sec} \quad \%OS = \exp\left(-\frac{\zeta\pi}{\sqrt{1-\zeta^2}}\right) \times 100\% = 77.95\%$$

$$\omega_n = 6.325 \text{ rad/s} \quad \zeta = 0.07906 \quad \omega_d = \omega_n \sqrt{1-\zeta^2} = 6.305 \text{ rad/s}$$

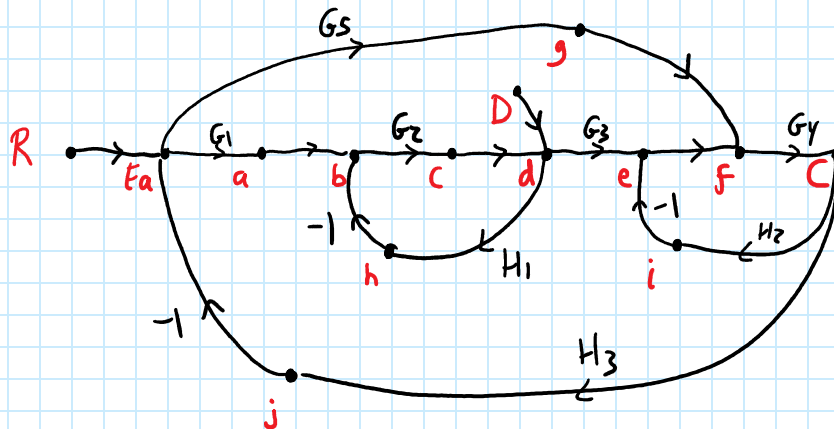
$$5) \quad c_{step}(\infty) = 0.8 \quad T_s = 5 \text{ sec} \quad \%OS = 10\%$$

$$\zeta = \frac{-\ln(\%OS/100)}{\sqrt{\pi^2 + \ln^2(\%OS/100)}} = 0.5912 \quad T_s = \frac{4}{\zeta\omega_n} \quad \omega_n = \frac{4}{(5 \times 0.5912)} = 1.353 \quad \omega_d = \omega_n \sqrt{1-\zeta^2} = 1.0914$$

$$s = -\zeta\omega_n \pm j\omega_d = -0.8 \pm j1.0914$$



(1)



$$(2) R \rightarrow C: T_1: E_a a b c d e f C = G_1 G_2 G_3 G_4 \quad L_1: b c d h b = -G_2 H_1 \quad L_3: E_a a b c d e f C j E_a = -G_1 G_2 G_3 H_3$$

$$T_2: R E_a g f C = G_4 G_5$$

$$L_2: e f C i e = -G_4 H_2 \quad L_4: E_a g f C j E_a = -G_1 G_5 H_3$$

$$\Delta = 1 - L_1 - L_2 - L_3 - L_4 + L_1 L_2 + L_1 L_4 \quad \Delta_1 = 1 \quad \Delta_2 = 1 - L_1$$

$$\frac{C}{R} = \frac{T_1 \Delta_1 + T_2 \Delta_2}{\Delta}$$

$$(3) D \rightarrow E_a: T_1 = D e f C j E_a = -G_3 G_4 H_3 \quad L_i \text{ \& } \Delta \text{ SAME AS ABOVE}$$

$$\Delta_1 = 1$$

$$\frac{E_a}{D} = \frac{T_1 \Delta_1}{\Delta}$$

3

$$(1) \quad \begin{array}{ccc} s^3 & b & d \\ s^2 & c & e \\ s^1 & \frac{be-cd}{c} & \\ s^0 & e & \end{array} \quad \begin{array}{l} b > 0 \\ c > 0 \\ \text{if } c > 0, cd > be \\ e > 0 \end{array}$$

$$s^2 \quad c \quad e \quad c > 0$$

$$s^1 \quad \frac{be-cd}{c} \quad \text{if } c > 0, cd > be$$

$$s^0 \quad e \quad e > 0$$

MARGINAL: i)  $e = 0$  &  $b, c, d > 0$  : POLE AT 0

ii)  $b, c, e > 0$  &  $be = cd$  : POLES AT  $\pm j\sqrt{e/c}$

iii)  $b, d > 0$   $c = e = 0$  : POLES AT 0,  $\pm j\sqrt{d/b}$

(2) POLE-ZERO CANCELLATIONS OR THINGS LIKE  $d = 0 \dots$

$$(3) \quad T = \frac{G}{1+G(D)} = \frac{K(s+1)}{s^4 + 14s^3 + 56s^2 + (34+K)s + 4K - 105} \quad \text{using } \frac{N}{D+N}$$

$$s^4 \quad 1 \quad 56 \quad 4K - 105$$

$$s^3 \quad 14 \quad 34+K$$

$$K < 750 \quad s^2 \quad \frac{750-K}{14} \quad 4K-105 \quad \rightarrow \quad \frac{(1)(34+K) - (14)(56)}{-14} = \frac{750-K}{14}$$

$$s^1 \quad \rightarrow \quad \frac{(14)(4K-105) - \left(\frac{750-K}{14}\right)(34+K)}{-\left(\frac{750-K}{14}\right)} \times 14$$

$$K > 26.25 \quad s^0 \quad 4K - 105$$

$$(14)(14)(4K-105) - (750-K)(34+K) < 0$$

$$784K - 20580 - 25500 - 750K + 34K + K^2 < 0$$

$$K^2 + 68K - 46080 < 0$$

$$\text{ROOTS: } \frac{-68 \pm \sqrt{68^2 - 4(-46080)}}{2}$$

$$\frac{-68 \pm \sqrt{46240 + 184320}}{2} = \frac{-68 \pm 217.34}{2}$$

$$K = 183.34, -251.34$$

$$(K+251.34)(K-183.34) < 0$$

$$-251.34 < K < 183.34$$

(b)  $K = 26.25$  POLE AT 0

$$K = 183.34 \rightarrow 40.48s^2 + 628.36 = 0, s = \pm j3.94 \text{ rad/s}$$

$$(c) \text{ TYPE 0 } K_p = \lim_{s \rightarrow 0} G = \frac{-4K}{105} \quad e_{\text{step}} = \frac{1}{1+K_p} = \frac{105}{105-4K}$$

MOST  
RESTRICTIVE

$$26.25 < K < 183.34$$

$$G = \frac{s^2 - 2s + 5}{s^3 + 6s^2 + 8s}$$

$$T = \frac{KG}{1+KG} = \frac{K(s^2 - 2s + 5)}{s^3 + (6+K)s^2 + (8-2K)s + 5K}$$

(1) MARGINAL GIVEN  $s$  IN DENOMINATOR ;  
NOTE: STEP RESPONSE IS UNBOUNDED!

(2)

$s^3$		$8-2K$
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$K > -6$

$s^2$	$6+K$	$5K$
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$s^1$

$\xrightarrow{\hspace{10em}} \frac{(1 \times 5K) - (6+K)(8-2K)}{-(6+K)}$

$\leftarrow K > -6$  FROM  $s^2$  LINE IS 0:

$(K > 0)$

$s^0$	$5K$
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$$0 < K < 3.14$$

$$5K - 48 + 12K - 8K + 2K^2 < 0$$

$$2K^2 + 9K - 48 < 0$$

$$K = \frac{-9 \pm \sqrt{81 - (4)(2)(-48)}}{(2)(2)}$$

$$= \frac{-9 \pm 21.56}{4} = -7.64, 3.14$$

(3) TYPE 1

(4)  $K_v$ , ramp

(5)  $K_v = \lim_{s \rightarrow 0} sG(K) = \frac{5K}{8}$

$$(K+7.64)(K-3.14) < 0$$

$$-7.64 < (K < 3.14)$$

(6)  $K = 3.14 \rightarrow K_v = 1.9625$   $e_{\text{ramp}} = \frac{1}{K_v} = 0.5096$

(7) (a)  $-0.421 \pm j0.797$

(b) UNDERDAMPED  $\rightarrow$  COMPLEX POLES w/ NEGATIVE REAL PARTS

$$T = \frac{K(s+5)}{s^3 + 8s^2 + (K-15)s + 5K-54}$$

$$G_{eg} = \frac{T}{1-T} = \frac{K(s+5)}{s^3 + 8s^2 - 15s - 54}$$

$$(1) \quad s^3 \quad 1 \quad K-15$$

$$s^2 \quad 8 \quad 5K-54$$

$$\textcircled{K > 22} \quad s^1 \quad \frac{(5K-54) - (8 \times (K-15))}{-8} > 0$$

$$K > 10.8 \quad s^0 \quad 5K-54$$

$$5K-54-8K+120 < 0$$

$$-3K + 66 < 0$$

$$K > 22$$

FLIP  $> 10 <$  FOR  $X-8$

(2) TYPE 0

(3)  $K_p$ , STEP

$$(4) \quad K_p = \lim_{s \rightarrow 0} G = \frac{-5K}{54} \quad e_{step} = \frac{1}{1+K_p} = \frac{54}{54-5K}$$

$$(5) \quad K=22; (6) \quad 8s^2 + 56 = 0 \quad s = \pm j\sqrt{7} = \pm j 2.646 \text{ rad/sec}$$