

$$1) A(s) = \frac{1}{s+2} + \frac{3}{s^2} + \frac{4(s+5)}{(s+5)^2 + 6^2}$$

$$2) \frac{5s+30}{s^2+14s+40} = \frac{5s+30}{(s+4)(s+10)} = \frac{\alpha}{s+4} + \frac{\beta}{s+10}$$

$|4^2 - 4 \cdot 40 > 0$: FACTOR

$$\alpha = \lim_{s \rightarrow -4} \frac{5s+30}{s+10} = \frac{10}{6} = \frac{5}{3} \quad \beta = \lim_{s \rightarrow -10} \frac{5s+30}{s+4} = \frac{-20}{-6} = \frac{10}{3}$$

$$b(t) = \left(\frac{5}{3} e^{-4t} + \frac{10}{3} e^{-10t} \right) u(t)$$

$$3) \frac{10s+300}{s^2+8s+116} = \frac{10s+300}{(s+4)^2+(10)^2} = \frac{\alpha(s+4) + \beta(10)}{(s+4)^2+(10)^2}$$

$$8^2 - 4 \cdot 116 < 0: \text{NOAT} \quad \begin{matrix} \alpha = 10 \\ \text{FROM } s \end{matrix} \quad \begin{matrix} 40 + 10\beta = 300 \\ \beta = 26 \end{matrix}$$

$$c(t) = e^{-4t} (10 \cos(10t) + 26 \sin(10t)) u(t)$$

$$4) X(s) = \frac{1}{s+3} \quad Y(s) = \frac{1}{(s+2)^2} \quad H = \frac{Y}{X} = \frac{s+3}{s^2+4s+4}$$

$$(s^2+4s+4)Y = (s+3)X$$

$$\ddot{y} + 4\dot{y} + 4y = \dot{x} + 3\underline{x}$$

$$5) (s^2 Y - sy(0^-) - \dot{y}(0^-)) + 4(sY - y(0^-)) + 3y = \frac{1}{s+5}$$

$$(s^2 + 4s + 3)Y - 2s - 0 - 8 = \frac{1}{s+5}$$

$$(s^2 + 4s + 3)Y = 2s + 8 + \frac{1}{s+5} = \frac{2s^2 + 18s + 41}{s+5}$$

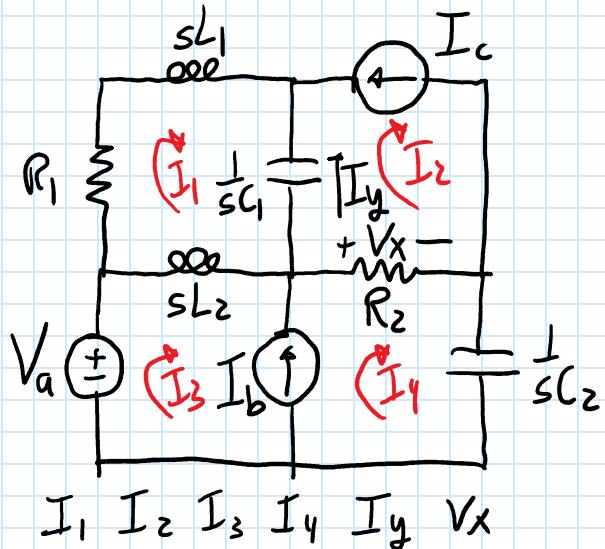
$$Y = \frac{2s^2 + 18s + 41}{(s^2 + 4s + 3)(s+5)} = \frac{2s^2 + 18s + 41}{(s+1)(s+3)(s+5)} = \frac{\alpha}{s+1} + \frac{\beta}{s+3} + \frac{\gamma}{s+5}$$

$$\alpha = \lim_{s \rightarrow -1} \frac{2s^2 + 18s + 41}{(s+3)(s+5)} = \frac{25}{(-2)(-8)} = \frac{25}{16} = \frac{25}{8}$$

$$\beta = \lim_{s \rightarrow -3} \frac{2s^2 + 18s + 41}{(s+1)(s+5)} = \frac{5}{(-2)(2)} = -\frac{5}{4}$$

$$\gamma = \lim_{s \rightarrow -5} \frac{2s^2 + 18s + 41}{(s+1)(s+3)} = \frac{1}{(-4)(-2)} = \frac{1}{8}$$

$$y(t) = \left(\frac{25}{8} e^{-t} - \frac{5}{4} e^{-3t} + \frac{1}{8} e^{-5t} \right) u(t)$$



$$KVL, l_1: R_1 I_1 + sL_1 (I_1 - I_2) + \frac{1}{sC_1} (I_1 - I_3) = 0$$

$$KVL, l_3: -V_a + sL_2 (I_3 - I_1) + R_2 (I_4 - I_2) + \frac{1}{sC_2} (I_4) = 0$$

$$SRC, I_b: I_b = I_4 - I_3$$

$$SRC, I_c: I_c = -I_2$$

$$MEAS, I_y: I_y = I_2 - I_1$$

$$MEAS, V_x: V_x = R_2 (I_4 - I_2)$$

$$(1) \quad Z_f = R_3 \quad Z_n = R_1 \quad H = -\frac{Z_f}{Z_n} = -\frac{R_3}{R_1}$$

$$(2) \quad Z_f = \frac{1}{sC_1} \parallel R_3 = \frac{\frac{R_3}{sC_1}}{\frac{R_3}{sC_1} + 1} = \frac{R_3}{sR_3C_1 + 1} \quad Z_n = R_1$$

$$H = -\frac{Z_f}{Z_n} = \frac{-R_3}{R_1(sR_3C_1 + 1)}$$

$$(3) \quad Z_{1f} = R_3 \quad Z_{1n} = R_1 + \frac{1}{sC_1} = \frac{sR_1C_1 + 1}{sC_1} \quad H_1 = -\frac{Z_f}{Z_n} = -\frac{sR_3C_1}{sR_1C_1 + 1}$$

$$Z_{2f} = \frac{1}{sC_2} \parallel R_6 = \frac{R_6}{sR_6C_2 + 1} \quad Z_{2n} = R_4 \quad H_2 = -\frac{Z_f}{Z_n} = -\frac{R_6}{R_4(sR_6C_2 + 1)}$$

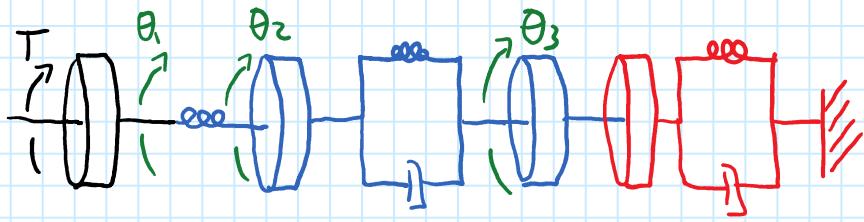
$$H = H_1 H_2 = \frac{sR_3 R_6 C_1}{(sR_1 C_1 + 1) R_4 (sR_6 C_2 + 1)}$$

$$M_1: (M_1 s^2 + (F_{V2} + F_{V3})s + K_2)X_1 - (F_{V2}s + K_2)X_2 - F_{V3}sX_3 = 0$$

$$M_2: -(F_{V2}s + K_2)X_1 + (M_2 s^2 + F_{V2}s + K_2)X_2 = 0$$

$$M_3: -F_{V3}sX_1 + (M_3 s^2 + (F_{V1} + F_{V3} + F_{V4})s + (K_1 + K_3))X_3 - (F_{V4}s + K_3)X_4 = 0$$

$$M_4: -(F_{V4}s + K_3)X_3 + (M_4 s^2 + F_{V4}s + K_3)X_4 = 0$$



$$\gamma_{21} = r_1 / r_2$$

$$r = r_g$$

$$J_1 \quad \gamma_{21}^2 K_1 \quad \gamma_{21}^2 J_2 \quad \gamma_{21}^2 K_2 \quad \gamma_{21}^2 J_g \quad \gamma_{21}^2 r^2 M \quad \gamma_{21}^2 r^2 K_L$$

$$\gamma_{21}^2 D \quad \gamma_{21}^2 r^2 F_v$$

$$1) (J_1 s^2 + \gamma_{21}^2 K_1) \Theta_1 - (\gamma_{21}^2 K_1) \Theta_2 = T$$

THESE ALL HAVE γ_{21}^2
HAVE γ_{21}^2
SO YOU COULD
FACTOR OUT

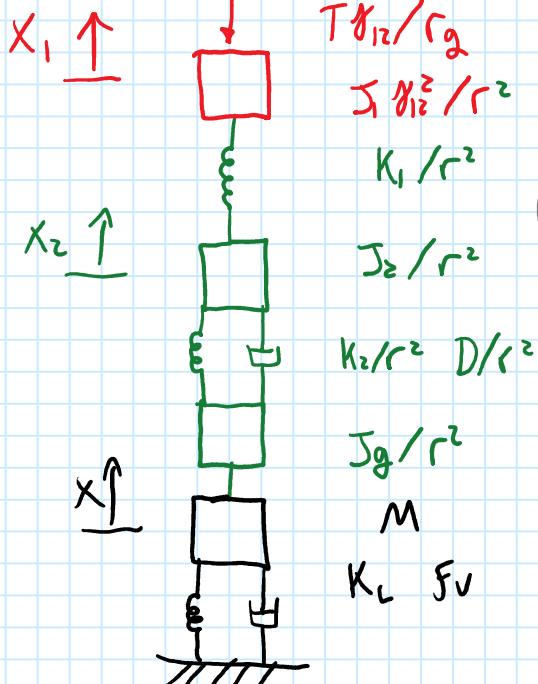
$$2) -(\gamma_{21}^2 K_1) \Theta_1 + (\gamma_{21}^2 J_2 s^2 + \gamma_{21}^2 D_s + (\gamma_{21}^2 K_1 + \gamma_{21}^2 K_2)) \Theta_2 - (\gamma_{21}^2 D + \gamma_{21}^2 K_2) \Theta_3 = 0$$

$$3) (\gamma_{21}^2 J_g + \gamma_{21}^2 r^2 M) s^2 + (\gamma_{21}^2 D + \gamma_{21}^2 r^2 F_v) s + (\gamma_{21}^2 K_2 + \gamma_{21}^2 r^2 K_L) \Theta_3 = 0$$

$$X = -r \Theta_{\text{gear}} \quad r_2 \Theta_{\text{gear}} \quad \Theta_{\text{gear}} r_2 = \Theta_3 r_1 \text{ so}$$

$$X = -\frac{r r_1 \Theta_3}{r_2}$$

ALT: X-LATIONAL FRAME



$$\gamma_{12} = r_2 / r_1$$

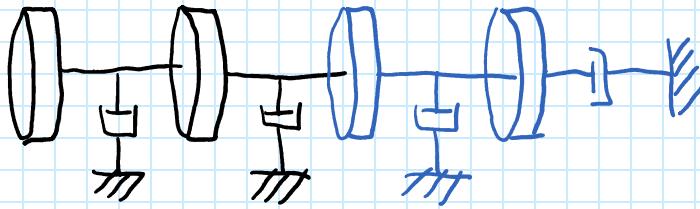
$$r = r_g$$

THESE ALL HAVE $\frac{1}{r^2}$ EXCEPT FOR

$$X_1: \left(\frac{J_1 \gamma_{12}^2}{r^2} s^2 + \frac{K_1}{r^2} \right) X_1 - \frac{K_1}{r^2} X_2 = -T \frac{\gamma_{12}}{r_g}$$

$$X_2: -\frac{K_1}{r^2} X_1 + \left(\frac{J_2 s^2}{r^2} + \frac{D_s}{r^2} + \left(\frac{K_1 + K_2}{r^2} \right) \right) X_2 - \left(\frac{D_s + K_2}{r^2} \right) X = 0$$

$$X_3: -\left(\frac{(D_s + K_2)}{r^2} \right) + \left(\left(\frac{J_g}{r^2} + M \right) s^2 + \left(\frac{D}{r^2} + F_v \right) s + \frac{K_2}{r^2} + K_L \right) X = 0$$

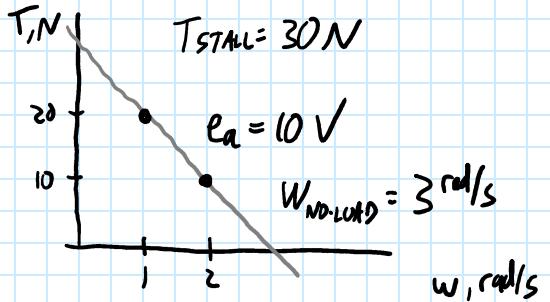


$$\gamma_{z1} = \frac{r_1}{r_2} = \frac{1}{2}$$

$$J_a \quad D_a \quad J_1 \quad D_1 \quad \gamma_{z1}^2 J_2 \quad \gamma_{z1}^2 D_2 \quad \gamma_{z1}^2 J_c \quad \gamma_{z1}^2 D_3$$

$$J_m = J_a + J_1 + \gamma_{z1}^2 J_2 + \gamma_{z1}^2 J_c = 1 + 2 + \frac{1}{4} \cdot 4 + \frac{1}{4} \cdot 100 = 29$$

$$D_m = D_a + D_1 + \gamma_{z1}^2 D_2 + \gamma_{z1}^2 D_c = .5 + .25 + \frac{1}{4} \cdot \frac{1}{2} + \frac{1}{4} \cdot 4 = 1.875$$



$$\frac{K_t}{R_a} = \frac{T_{\text{start}}}{e_a} = 3$$

$$K_b = \frac{e_a}{w_{\text{no-load}}} = \frac{10}{3}$$

$$\frac{\underline{H}_M}{E_a} = \frac{\frac{K_t}{R_a} \frac{1}{J_m}}{s(s + \frac{1}{J_m}(D_m + \frac{K_t K_b}{R_a}))} = \frac{\frac{3}{29}}{s(s + \frac{1}{29}(1.875 + \frac{3 \cdot 10}{3}))}$$

$$= \frac{0.1034}{s(s + 0.4095)} \quad \theta_c r_2 = \theta_m r_1, \quad \theta_c = \frac{\theta_m r_1}{r_2} = \frac{\theta_m}{2}$$

$$\frac{\underline{H}_C}{E_a} = \frac{0.0517}{s(s + 0.4095)} \quad \frac{\underline{S}_c}{E_a} = s \frac{\underline{\theta}_c}{E_a} = \frac{0.0517}{(s + 0.4095)}$$

$$\text{IF } E_a = \frac{10}{s} \quad \underline{S}_c = \frac{0.0517}{(s(s + 0.4095))} = -\frac{\alpha}{s} + \frac{\beta}{s + 0.4095}$$

$$\alpha = \lim_{s \rightarrow 0} \frac{0.0517}{s + 0.4095} = 0.1263 \quad \beta = \lim_{s \rightarrow -0.4095} \frac{0.0517}{s} = -0.1263$$

$$w(t) = (0.1263 - 0.1263 e^{-0.4095 t}) u(t)$$