Duke University
Edmund T. Pratt, Jr. School of Engineering

# Controls Summer 2020
# Test III

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Name (please print):

NetID (please print): Start Time: End Time:

Submitting your work for a grade implies agreement with the following: In keeping with the Community Standard, I have neither provided nor received any assistance on this test. I understand if it is later determined that I gave or received assistance, I will be brought before the Undergraduate Conduct Board and, if found responsible for academic dishonesty or academic contempt, fail the class. I also understand that I am not allowed to communicate with anyone except the instructor about any aspect of this test until the instructor announces it is allowed. I understand if it is later determined that I did communicate with another person about the test before the instructor said it was allowed, I will be brought before the Undergraduate Conduct Board and, if found responsible for academic dishonesty or academic contempt, fail the class.

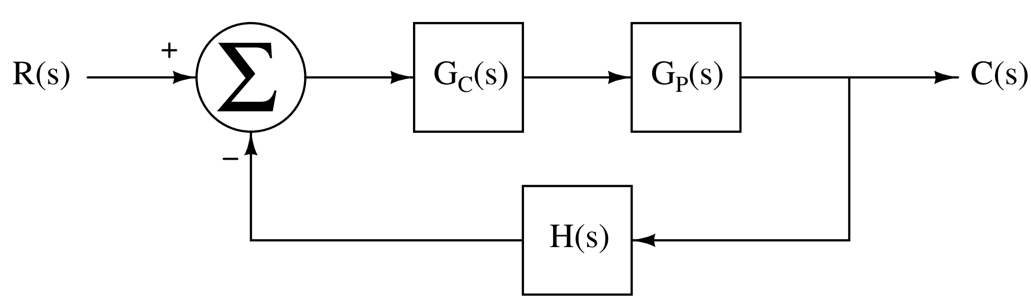
## Instructions

The test is open book, open class notes (yours and mine), open Sakai page for this class, and open Pundit. You are also allowed to use MATLAB. No other resources are allowed. If you have a question about whether something is allowed, ask the instructor. Be sure that you are in a place where you can work undisturbed for the duration of the test. If a situation arises that disrupts your work, let the instructor know - you will be allowed to pause the clock and resume work later. The Start Time above should be when you first started working on the test (after saving it, printing it out, etc.) and the End Time should be when you stopped doing work on the test and started working on scanning / photographing and uploading it.

**Please be sure to clearly indicate where each answer for each part of each problem is.** You will be turning in several files. There will be a single PDF of work that will go to Gradescope as well as several other files that will go to Sakai. More instructions for this may be found on the next page of the test. As with previous tests, you will upload the PDF to Sakai first - this will stop the test timer. **Do not make any changes to your document once you have uploaded it to Sakai.** Carefully scan or photograph the test pages in order (with any additional pages properly labeled and after all the original test pages) and make a PDF of the scans / photographs. When you upload the PDF to Gradescope, you will also need to indicate where the answers for each problems are. Please do not include this cover page as a part of any of the assigned pages. After that, you will be putting files in specific sections of your Sakai drop box. That process is discussed on the next page.

Note that there may be people taking the test before or after you, so you are not allowed to talk about the test - even to people outside of this class - until I send along the OK. This includes talking about the specific problem types, how long it took you, how hard you thought it was - really anything. Please maintain the integrity of this test. You are allowed to study with anyone so long as no one in the group has started the test yet.

### General System Model



# Instructions for Submitting Work

Each problem has some work to be done by hand. All the work for those problems should be put into a single PDF document and uploaded to Sakai through the test portal and then to Gradescope. As with previous tests, indicate the pages for each problem - do not include the cover page or this page when assigning pages in Gradescope. The specific parts that should be detailed in this PDF are:

- All parts of Problem I
- Parts (1)-(4) of Problem II
- Parts (1)-(4) of Problem III

There are also some design system discussion questions. The answers to those can either go in this main PDF or they can be located in system-design-specific files described below. Just be sure to clearly indicate which discussion question you are answering by providing the problem number, question number, and prompt letter.

Other files will go in your Sakai Drop Box. When ready, go to Sakai and open the Drop Box section via the link on the left. Click on the Actions button and create a folder called `Test 3`. Click on Test 3 to open that folder, then within it create folders `Problem 2` and `Problem 3`. The files that should go in each folder are as follows:

- Problem 2:
  - A document showing the SISOtool generated root locus figure and step response figure (including peak characteristics, settling time characteristics, and steady state characteristics) along with the compensator, the location of all system zeros, and the location of all system poles for "SYSTEM 1." Also, provide answers to discussion questions (5b) and (5c) in this document or in the main PDF. For these and all other discussion questions, clearly indicate what you are answering (for instance, write II(5a) before your answer).
  - A document (can be in the same file above) showing the SISOtool generated root locus figure and step response figure (including peak characteristics, settling time characteristics, and steady state characteristics) along with the compensator, the location of all system zeros, and the location of all system poles for "SYSTEM 2." Also, provide answers to discussion questions (6b) and (6c) in this document or in the main PDF.
  - A document (can be in the same file above) showing the SISOtool generated root locus figure and step response figure (including peak characteristics, settling time characteristics, and steady state characteristics) along with the compensator, the location of all system zeros, and the location of all system poles for "SYSTEM 3." Also, provide answers to discussion questions (7b) and (7c) in this document or in the main PDF.
  - Saved SISOtool sessions (.mat files) for each of the three systems; be sure to save them after you have satisfied the current design criteria but before you move on to the next design criteria.

- Problem 3:
  - A document showing the SISOtool generated root locus figure and step response figure (including peak characteristics, settling time characteristics, and steady state characteristics) along with the compensator, the location of all system zeros, and the location of all system poles for "SYSTEM 1." Also, provide answers to discussion questions (5b) and (5c) in this document or in the main PDF.
  - A document (can be in the same file above) showing the SISOtool generated root locus figure and step response figure (including peak characteristics, settling time characteristics, and steady state characteristics) along with the compensator, the location of all system zeros, and the location of all system poles for "SYSTEM 2." Also, provide answers to discussion question (6b) in this document or in the main PDF.
  - A document (can be in the same file above) showing the SISOtool generated root locus figure and step response figure (including peak characteristics, settling time characteristics, and steady state characteristics) along with the compensator, the location of all system zeros, and the location of all system poles for "SYSTEM 3." Also, provide answers to discussion question (7b) in this document or in the main PDF.
  - Saved SISOtool sessions (.mat files) for each of the three systems; be sure to save them after you have satisfied the current design criteria but before you move on to the next design criteria.

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## Problem I: [30 *pts.*] Root Locus

For the following problems, you are given a system with a gain controller $G_C = K$, a plant $G_P$, and an output transducer $H$. You are also given the overall transfer function for each system, $T$, which includes the gain controller. For each, you must:

**(1)** Determine and report the range of $K$ for stability of the system.

**(2)** Make an accurate sketch of the root locus for positive gains $K$. You must clearly calculate any $j\omega$ crossings and the $K$ values at which they occur, asymptote centers, and asymptote angles. For break-in or break-out points, the potential $\sigma$ values and their corresponding $K$ values are given in the table.

**(3)** Determine and report the locations of any closed-loop zeros for the system.

**(4)** For each system, determine and report the $K$ values where the dominant poles are underdamped.

**(5)** For each system, determine and report the $K$ value or values you believe will give the system its fastest settling time then calculate an approximation for that settling time. How accurate do you believe that approximation might be and why?

| | System 1 | System 2 | System 3 |
|---|---|---|---|
| $G_P$ | $\frac{s+3}{(s-2)(s+1)(s+5)}$ | $\frac{1}{(s-1)(s+4)}$ | $\frac{1}{s^2-6s+13}$ |
| $H$ | $1$ | $\frac{1}{s+5}$ | $s^2+11s+30$ |
| $T$ | $\frac{K(s+3)}{s^3+4s^2+(K-7)s+3K-10}$ | $\frac{K(s+5)}{s^3+8s^2+11s+K-20}$ | $\frac{K}{(K+1)s^2+(11K-6)s+30K+13}$ |
| $\sigma, \frac{dK(\sigma)}{dt}=0$ | $0.377$ | $-4.52, -0.811$ | $-5.47, 3.47$ |
| $K, \frac{dK(\sigma)}{dt}=0$ | $3.56$ | $-1.38, 24.2$ | $304, -0.0526$ |

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## Problem II: [38 pts.] System Compensation the First

Assume a unity feedback system with input $r(t)$, output $c(t)$, a gain controller $G_{C1} = K$, and a plant with a transfer function equal to:

$$G_P = \frac{1}{s(s+3)(s+7)} = \frac{1}{s^3 + 10s^2 + 21s}$$

Note that a break-in/break-out point analysis reveals that

$$\frac{dK(\sigma)}{d\sigma} = 0 \text{ when } \sigma = -5.36, -1.31 \text{ where } K = -20.7, 12.6$$

while calculating the overall transfer function with a gain controller $G_{\text{C1}} = K$ yields:

$$T = \frac{K}{s^3 + 10s^2 + 21s + K}$$

Furthermore, creating a Routh Array of the overall transfer function yields:

$$\begin{bmatrix} 1 & 21 & s^3 \\ 10 & K & s^2 \\ 21 - \frac{K}{10} & 0 & s \\ K & 0 & 1 \end{bmatrix}$$

**(1)** What is the range of stability for a gain controller? Include both positive and negative values if they are stable.

**(2)** Sketch an accurate representation of the root locus plot for this system with gain controller $G_{\text{C1}}$ for positive gains. You must clearly calculate any $j\omega$ crossings, asymptote centers, asymptote angles, and break-in/break-out locations.

**(3)** Can this system's dominant poles ever be critically damped with positive gain? If not, state why you believe that. If so, determine the location of the dominant poles and the value for the gain to make the system's dominant poles critically damped with positive gain.

**(4)** Assuming positive gain, what is the shortest settling time for this system? Why do you believe that?

**(5)** SYSTEM 1:

- *(a)* Using SISOtool, generate the root locus for the system with gain control. Report the gain and the location of the overall system zeros and poles when the dominant poles represent an underdamped system with 20% overshoot. Include a picture of the root locus at this design point as well as a picture of the step response with the peak response, settling time, and steady state characteristics indicated in the plot. Save the session with a name that clearly indicates it is the session for Test 3 Problem 2 System 1.
- *(b)* Given the location of the dominant poles, what would you determine as the approximate settling time? Given the location of the poles and zeros, how accurate do you believe that approximation should be and why? How accurate is it?
- *(c)* From a steady-state error perspective, what type of system does this represent with controller $G_{C1}$ in its current configuration? What is the value of the appropriate static error constant? What type of unit input would produce a finite steady-state error? What would that error be?

**(6)** SYSTEM 2:

*(a)* Using SISOtool, design an active controller $G_{C2}$ which will reduce the approximate settling time to 1.5 sec (according to SISOtool) with the same 20% overshoot. Report the compensator and the location of the overall system zeros and poles when the design criteria are satisfied. Include a picture of the root locus at this design point as well as a picture of the step response with the peak response, settling time, and steady state characteristics indicated in the plot. Save the session with a name that clearly indicates it is the session for Test 3 Problem 2 System 2.

*(b)* Given the location of the dominant poles, what would you determine as the approximate settling time? Given the location of the poles and zeros, how accurate do you believe that approximation should be and why? How accurate is it?

*(c)* From a steady-state error perspective, what type of system does this represent with controller $G_{C2}$ in its current configuration? What is the value of the appropriate static error constant? What type of unit input would produce a finite steady-state error? What would that error be?

**(7)** SYSTEM 3:

*(a)* Using SISOtool, design a passive controller $G_{C3}$ which will maintain the approximate settling time of 1.5 sec and the 20% overshoot, but which has an excellent chance of the approximations being the most accurate. Report the compensator and the location of the overall system zeros and poles when the design criteria are satisfied. Include a picture of the root locus at this design point as well as a picture of the step response with the peak response, settling time, and steady state characteristics indicated in the plot. Save the session with a name that clearly indicates it is the session for Test 3 Problem 2 System 3.

*(b)* Given the location of the dominant poles, what would you determine as the approximate settling time? Given the location of the poles and zeros, how accurate do you believe that approximation should be and why? How accurate is it?

*(c)* From a steady-state error perspective, what type of system does this represent with controller $G_{C3}$ in its current configuration? What is the value of the appropriate static error constant? What type of unit input would produce a finite steady-state error? What would that error be?

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## Problem III: [32 pts.] System Compensation the Second

Assume a unity feedback system with input $r(t)$, output $c(t)$, a gain controller $G_{C1} = K$, and a plant with a transfer function equal to:

$$G_P = \frac{(s+8)(s+10)}{(s+1)(s+3)} = \frac{s^2+18s+80}{s^2+4s+3}$$

Note that a break-in/break-out point analysis reveals that

$$\frac{dK(\sigma)}{d\sigma} = 0 \text{ when } \sigma = -8.85, -2.15 \text{ where } K = 47.0, 0.0213$$

while calculating the overall transfer function with a gain controller $G_{C1} = K$ yields:

$$T = \frac{K(s^2+18s+80)}{(K+1)s^2+(18K+4)s+80K+3}$$

**(1)** What is the range of stability for a gain controller? Include both positive and negative values if they are stable.

**(2)** Sketch an accurate representation of the root locus plot for this system with gain controller $G_{C1}$ for positive gains. You must clearly calculate any $j\omega$ crossings, asymptote centers, asymptote angles, and break-in/break-out locations.

**(3)** Can this system's dominant poles ever be critically damped with positive gain? If not, state why you believe that. If so, determine the location of the dominant poles and the value for the each positive gain that makes the system's dominant poles critically damped with positive gain.

**(4)** Assuming positive gain, what is the shortest settling time for this system? Why do you believe that?

**(5)** SYSTEM 1:

*(a)* Using SISOtool, generate the root locus for the system with gain control. Report the gain and the location of the overall system zeros and poles when the dominant poles represent an underdamped system with a 1.5 sec settling time. Include a picture of the root locus at this design point as well as a picture of the step response with the peak response, settling time, and steady state characteristics indicated in the plot. Save the session with a name that clearly indicates it is the session for Test 3 Problem 3 System 1.

*(b)* Given the location of the dominant poles, what would you determine as the approximate percent overshoot? Given the location of the poles and zeros, how accurate do you believe that approximation should be and why? How accurate is it?

*(c)* From a steady-state error perspective, what type of system does this represent with controller $G_{C1}$ in its current configuration? What is the value of the appropriate static error constant? What type of unit input would produce a finite steady-state error? What would that error be?

**(6)** SYSTEM 2:

*(a)* Using SISOtool, design a passive controller $G_{C2}$ which will reduce the finite steady state error by a factor of 3 from the value given above without altering the location of the second order poles from the system above. In your compensator design, place any required poles or zeros such that the furthest distance either item is away from the origin is 5% of the location of the real part of the 2nd order poles. That is, if the second order poles for SYSTEM 1 were at $-4 \pm j6$ (they aren't), the pole or zero furthest from the origin for this compensator would be at -0.2. Report the compensator and the location of the overall system zeros and poles when the design criteria are satisfied. Include a picture of the root locus at this design point as well as a picture of the step response with the peak response, settling time, and steady state characteristics indicated in the plot. Save the session with a name that clearly indicates it is the session for Test 3 Problem 3 System 2.

*(b)* From a steady-state error perspective, what type of system does this represent with controller $G_{C2}$ in its current configuration? What is the value of the appropriate static error constant? What type of unit input would produce a finite steady-state error? What would that error be?

**(7)** SYSTEM 3:

*(a)* Using SISOtool, design an active controller $G_{C3}$ which will leave the 2nd order poles very close to their current location but which will increase the system type from a steady state error perspective. In your compensator design, place any required poles or zeros such that the furthest distance either item is away from the origin is 5% of the location of the real part of the 2nd order poles. That is, if the second order poles for SYSTEM 1 were at $-4 \pm j6$ (they aren't), the pole or zero furthest from the origin for this compensator would be at -0.2. Report the compensator and the location of the overall system zeros and poles when the design criteria are satisfied. Include a picture of the root locus at this design point as well as a picture of the step response with the peak response, settling time, and steady state characteristics indicated in the plot. Save the session with a name that clearly indicates it is the session for Test 3 Problem 3 System 3.

*(b)* From a steady-state error perspective, what type of system does this represent with controller $G_{C3}$ in its current configuration? What is the value of the appropriate static error constant? What type of unit input would produce a finite steady-state error? What would that error be?