Duke University
Edmund T. Pratt, Jr. School of Engineering

Controls Summer 2020
Test II

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Name (please print):

NetID (please print): Start Time: End Time:

Submitting your work for a grade implies agreement with the following: In keeping with the Community Standard, I have neither provided nor received any assistance on this test. I understand if it is later determined that I gave or received assistance, I will be brought before the Undergraduate Conduct Board and, if found responsible for academic dishonesty or academic contempt, fail the class. I also understand that I am not allowed to communicate with anyone except the instructor about any aspect of this test until the instructor announces it is allowed. I understand if it is later determined that I did communicate with another person about the test before the instructor said it was allowed, I will be brought before the Undergraduate Conduct Board and, if found responsible for academic dishonesty or academic contempt, fail the class.

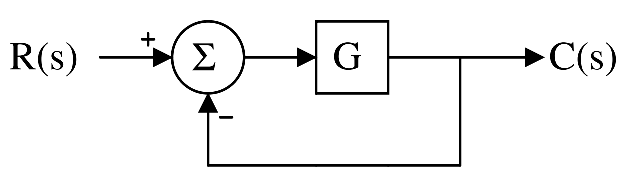
Instructions

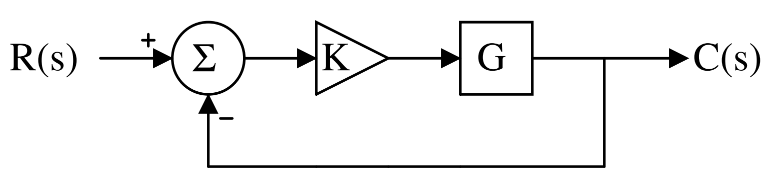
The test is open book, open class notes (yours and mine), open Sakai page for this class, and open Pundit. No other resources are allowed. If you have a question about whether something is allowed, ask the instructor. Be sure that you are in a place where you can work undisturbed for the duration of the test. If a situation arises that disrupts your work, let the instructor know - you will be allowed to pause the clock and resume work later. The Start Time above should be when you first started working on the test (after saving it, printing it out, etc.) and the End Time should be when you stopped doing work on the test and started working on scanning / photographing and uploading it.

Please be sure to clearly indicate where each answer for each part of each problem is. You will be turning in your work to both Sakai and Gradescope as a single PDF. Upload the PDF to Sakai first - this will stop the test timer. **Do not make any changes to your document once you have uploaded it to Sakai.** Carefully scan or photograph the test pages in order (with any additional pages properly labeled and after all the original test pages) and make a PDF of the scans / photographs. When you upload the PDF to Gradescope, you will also need to indicate where the answers for each problems are. Please do not include this cover page as a part of any of the assigned pages.

Note that there may be people taking the test before or after you, so you are not allowed to talk about the test - even to people outside of this class - until I send along the OK. This includes talking about the specific problem types, how long it took you, how hard you thought it was - really anything. Please maintain the integrity of this test. You are allowed to study with anyone so long as no one in the group has started the test yet.

Note: in all problems, $u(t)$ is the unit step and $\delta(t)$ is the unit impulse. Also, the following represent the block diagrams for unity feedback systems and unity feedback systems with cascaded proportional (gain) control, respectively:





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Problem I: [24 *pts.*] Transient and Steady-State Characteristics

While your work for these problems may go on other pages, your final answers should go in the space provided below the prompts.

(1) Determine the steady-state value and the settling time of the step response for a system that has an overall transfer function of:

$$T_{\mathrm{w}}(s) = \frac{8}{s+10}$$

(2) Determine the steady-state value and the settling time of the step response for a unity feedback system that has a forward transfer function of:

$$G_{\mathrm{x}}(s) = \frac{2}{s+1}$$

(3) Determine the steady-state value, settling time, and %OS of the step response as well as the natural frequency, damping ratio, and damped frequency for a system that has an overall transfer function of:

$$T_{\mathrm{y}}(s) = \frac{15}{s^2+7s+23}$$

$c_{\mathrm{step}}(\infty) =$ $\quad$ $T_S =$ $\quad$ $\%OS =$

$\omega_n =$ $\quad$ $\zeta =$ $\quad$ $\omega_d =$

(4) Determine the steady-state value, settling time, and %OS of the step response as well as the natural frequency, damping ratio, and damped frequency for a unity feedback system that has a forward transfer function of:

$$G_{\mathrm{z}}(s) = \frac{50}{s^2+10s+5}$$

$c_{\mathrm{step}}(\infty) =$ $\quad$ $T_S =$ $\quad$ $\%OS =$

$\omega_n =$ $\quad$ $\zeta =$ $\quad$ $\omega_d =$

(5) A first-order system is found to have a step response with a steady state error of 20% and a rise time of 4 seconds. What is that system's overall transfer function?

(6) An underdamped second-order system is found to have a step response with a steady state value of 2; the transient includes a maximum 30% overshoot that occurs 2 seconds after the step input begins. What is that system's overall transfer function?

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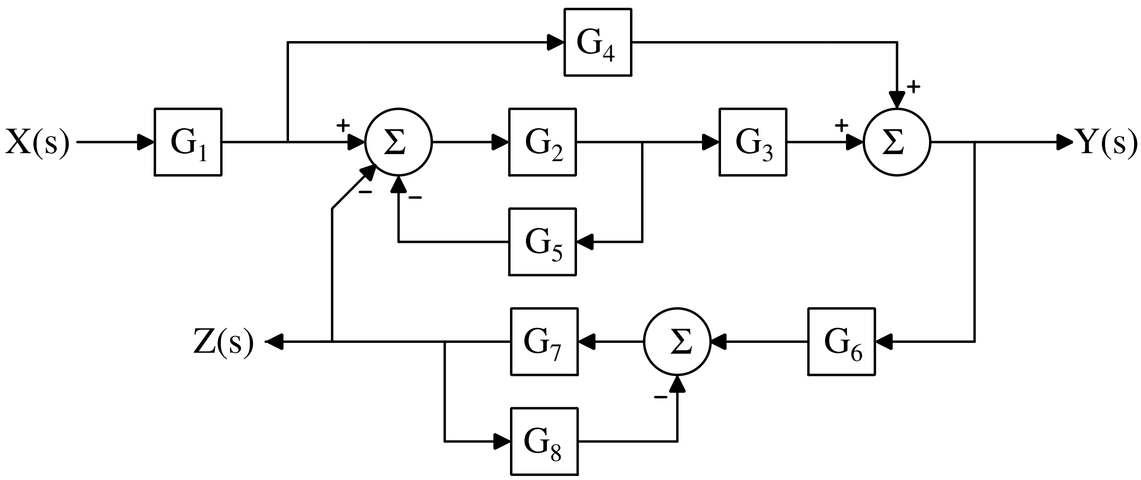
Problem II: [16 *pts.*] System Simplification

Given the system below:



(1) Clearly draw a signal flow diagram for the system. Be sure to indicate where each node is on the diagram - *especially* **X**, **Y**, and **Z**.

(2) Clearly use Mason's Rule to determine the overall transfer function $T_{\mathrm{y}}(s) = Y(s)/X(s)$. Note that once you define components in terms of the individual transfer functions, you do *not* need to simplify nor do you need to substitute them into the final result.

(3) Clearly use Mason's Rule to determine the transfer function $T_{\mathrm{z}}(s) = Z(s)/X(s)$. Note that once you define components in terms of the individual transfer functions, you do *not* need to simplify nor do you need to substitute them into the final result. If you re-use any components defined for $T_{\mathrm{y}}(s)$ above, simply state that.

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Problem III: [20 *pts.*] Stability and Steady State Analysis

While your work for these problems - including the Routh arrays - may go on other pages, your final answers should go in the space provided below the prompts. **You must clearly build and interpret Routh arrays to support your conclusions for the following problems** in order to get any credit.

(1) A system has an overall transfer function of:

$$T_{\mathrm{a}}(s) = \frac{100}{2s^4 + s^3 + 2s^2 - 12s - 16}$$

How many left, right, and $j\omega$ poles does the overall system have?

(2) A system has an overall transfer function of:

$$T_{\mathrm{b}}(s) = \frac{100}{2s^4 + 4s^3 + s^2 + 2s + 10}$$

How many left, right, and $j\omega$ poles does the overall system have?

(3) A system has an overall transfer function of:

$$T_{\mathrm{c}}(s) = \frac{100}{s^5 + 3s^4 + s^3 + 3s^2 - 2s - 6}$$

How many left, right, and $j\omega$ poles does the overall system have?

(4) A unity feedback system has a forward path transfer function of:

$$G_{\mathrm{d}}(s) = \frac{K(s+8)}{s^3 + 7s^2 - 6s - 72}$$

(a) Determine the conditions on K for the system to be stable.

(b) Are there one or more non-trivial values of K which would cause marginal stability? If so, find them and determine the frequency of oscillation when the system is marginally stable.

(c) For values of K that keep the system stable, what is the system type from a steady-state error perspective?

(d) Given that system type, determine the value for the appropriate finite (non-zero, non-infinite) static error constant and the steady state position error as a function of K.

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Problem IV: [16 *pts.*] System Analysis I

Be sure to clearly indicate which part of the problem you are answering by writing the appropriate section/subsection/subsubsection before your answer. For example, "(IV.2): The values of K that keep the system stable are..." A unity feedback system has a gain controller K in cascade with a plant:

$$G(s) = \frac{s^2 + 18s + 80}{s^2 - 12s + 27}$$

(1) For what values of K will the overall system be stable? Justify your answer.

(2) For values of K that keep the system stable, what system type does this represent from a steady-state error perspective?

(3) Circle the appropriate statement or statements for each of the static error constants and line through the incorrect statements in the table below:

K_p	K_v	K_a
is 0	is 0	is 0
is ∞	is ∞	is ∞
is neither 0 nor ∞	is neither 0 nor ∞	is neither 0 nor ∞

(4) For any of the above where you circled "is neither 0 nor ∞" calculate the finite value of the appropriate static error constant as a function of K.

(5) For any of the above where you circled "is neither 0 nor ∞" find the gain K that would produce a steady-state position error of exactly 20%.

(6) Assuming a position error of exactly 20%, what input $r(t)$ would produce a steady-state position error value of $e(\infty) = 10$?

Problem V: [24 *pts.*] System Analysis II

Be sure to clearly indicate which part of the problem you are answering by writing the appropriate section/subsection/subsub-section before your answer. For example, "(V.7): The input that produces $e(\infty) = 10$ is ..." A system with gain control has an overall transfer function T of:

$$T(s) = \frac{K(s+4)}{s^4 + 16s^3 + 81s^2 + (K+126)s + 4K}$$

Analysis of this system's Routh array reveals that the system is stable for $0 < K < 394.08$. Here are the normalized entries for the Routh array:

$$\begin{bmatrix} s^4 & 1 & 81 & 4K \\ s^3 & 16 & K+126 & 0 \\ s^2 & 1170-K & 64K & 0 \\ s^1 & \frac{K^2-20K-147420}{K-1170} & 0 & 0 \\ s^0 & K & 0 & 0 \end{bmatrix}$$

(1) For values of K that keep the system stable, what system type does this represent from a steady-state error perspective?

(2) For what value of K will the overall system by non-trivially marginally stable?

(3) For the value of K that makes the system non-trivially marginally stable, what frequency for the input would lead to an unbounded output?

(4) Circle the appropriate statement or statements for each of the static error constants and line through the incorrect statements in the table below:

K_p	K_v	K_a
is 0	is 0	is 0
is ∞	is ∞	is ∞
is neither 0 nor ∞	is neither 0 nor ∞	is neither 0 nor ∞

(5) For any of the above where you circled "is neither 0 nor ∞" calculate the finite value of the appropriate static error constant as a function of K.

(6) For any of the above where you circled "is neither 0 nor ∞" find the gain K that would produce a steady-state position error of exactly 20%.

(7) Assuming a position error of exactly 20%, what input $r(t)$ would produce a steady-state position error value of $e(\infty) = 10$?

(8) At a particular K value in the stable range, the overall system poles and zeros[1] are at:

$$\begin{aligned} &\text{zeros} : -4.0 \\ &\text{poles} : -10.70, -4.11, -0.58 \pm 4.4j \end{aligned}$$

(a) Given those poles and zeros, approximately what are the settling time and the %OS for the system?

(b) For the approximations above, how confident are you that they would stand up in comparison with values obtained through numerical simulation? Why do you say that?

[1] The book only uses one e in zeros, alas