Duke University
Edmund T. Pratt, Jr. School of Engineering

Controls Summer 2020
Test I

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Name (please print):

NetID (please print): Start Time: End Time:

Submitting your work for a grade implies agreement with the following: In keeping with the Community Standard, I have neither provided nor received any assistance on this test. I understand if it is later determined that I gave or received assistance, I will be brought before the Undergraduate Conduct Board and, if found responsible for academic dishonesty or academic contempt, fail the class. I also understand that I am not allowed to communicate with anyone except the instructor about any aspect of this test until the instructor announces it is allowed. I understand if it is later determined that I did communicate with another person about the test before the instructor said it was allowed, I will be brought before the Undergraduate Conduct Board and, if found responsible for academic dishonesty or academic contempt, fail the class.

Instructions

The test is open book, open class notes (yours and mine), open Sakai page for this class, and open Pundit. No other resources are allowed. If you have a question about whether something is allowed, ask the instructor. Be sure that you are in a place where you can work undisturbed for the duration of the test. If a situation arises that disrupts your work, let the instructor know - you will be allowed to pause the clock and resume work later. The Start Time above should be when you first started working on the test (after saving it, printing it out, etc.) and the End Time should be when you stopped doing work on the test and started working on scanning / photographing and uploading it.

Please be sure to clearly indicate where each answer for each part of each problem is. You will be turning in your work to both Sakai and Gradescope as a single PDF. Upload the PDF to Sakai first - this will stop the test timer. **Do not make any changes to your document once you have uploaded it to Sakai.** Carefully scan or photograph the test pages in order (with any additional pages properly labeled and after all the original test pages) and make a PDF of the scans / photographs. When you upload the PDF to Gradescope, you will also need to indicate where the answers for each problems are. Please do not include this cover page as a part of any of the assigned pages.

Note that there may be people taking the test before or after you, so you are not allowed to talk about the test - even to people outside of this class - until I send along the OK. This includes talking about the specific problem types, how long it took you, how hard you thought it was - really anything. Please maintain the integrity of this test. You are allowed to study with anyone so long as no one in the group has started the test yet.

Note: in all problems, $u(t)$ is the unit step and $\delta(t)$ is the unit impulse.

Problem I: [25 pts.] Math

(1) Find the Laplace transform $\mathcal{A}(s)$ of the signal $a(t)$ given:

$$a(t) = 3\left(1 - e^{-2(t-1)}\right) u(t-1)$$

(2) Find the signal $b(t)$ given its Laplace transform $\mathcal{B}(s)$:

$$\mathcal{B}(s) = \frac{20 + 30e^{-4s}}{s^2 + 12s + 35}$$

(3) Find the signal $c(t)$ given its Laplace transform $\mathcal{C}(s)$:

$$\mathcal{C}(s) = \frac{50s}{s^2 + 12s + 40}$$

(4) If the step response $s_r(t)$ of an LTI system with an input named $x(t)$ and an output named $y(t)$ is:

$$s_r(t) = \left(\frac{1}{2} - \frac{5}{2}e^{-4t}\right) u(t)$$

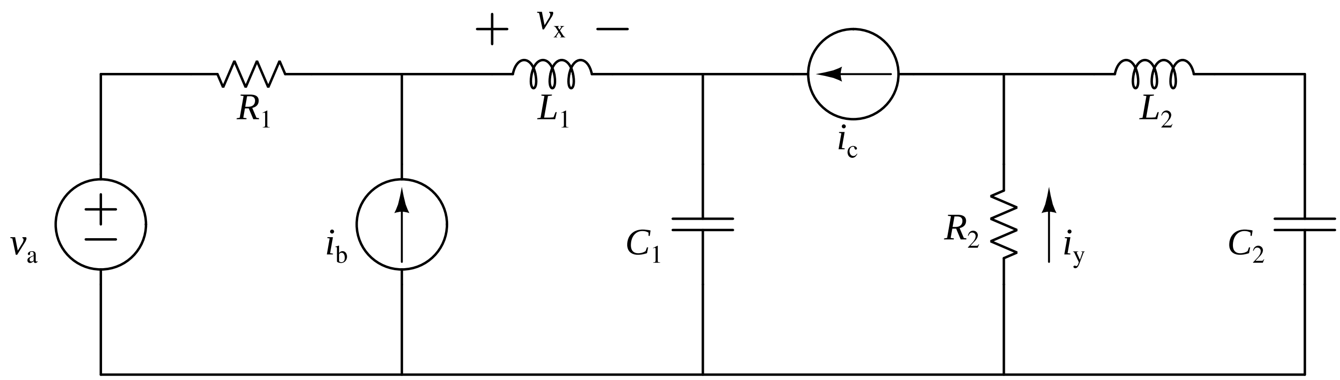
determine the differential equation relating $y(t)$ and its derivatives to $x(t)$ and its derivatives.

(5) Clearly use Laplace techniques to solve the following differential equation for $y(t), t > 0$, assuming the given initial conditions:

$$\frac{d^2y(t)}{dt^2} + 6\frac{dy(t)}{dt} + 9y(t) = e^{-2t}$$
$$y(0^-) = 7$$
$$\dot{y}(0^-) = 0$$

Problem II: [14 pts.] Electronics

Given the following electrical system:

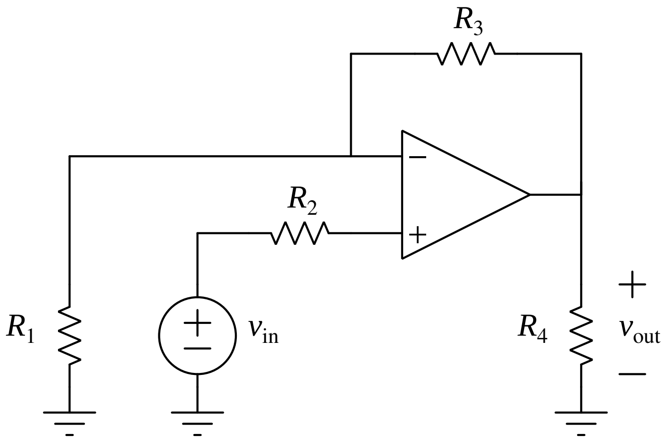


and assuming that the values for the passive elements (L_1, L_2, R_1, R_2, C_1, C_2), and the values for the independent sources (v_a, i_b, i_c) are known, *clearly* demonstrate the use of the Mesh Current Method in the frequency domain to label unknowns for the circuit and to determine a complete set of equations in the frequency domain that could be used to solve for these unknowns, including the voltage measurement $\mathbb{V}_x$ and the current measurement $\mathbb{I}_y$. List the set of unknowns you believe your equations will find. You do not need to arrange the equations in matrix format nor do you need to solve the equations.

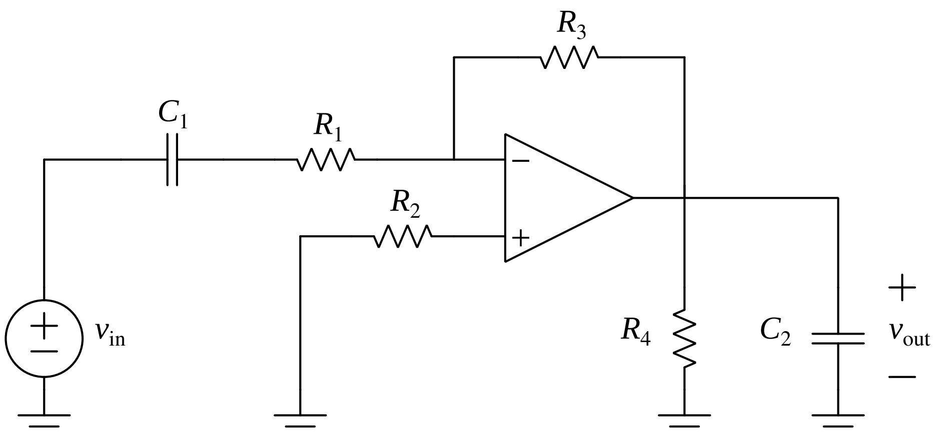
Unknowns:

Problem III: [18 pts.] Amplifiers

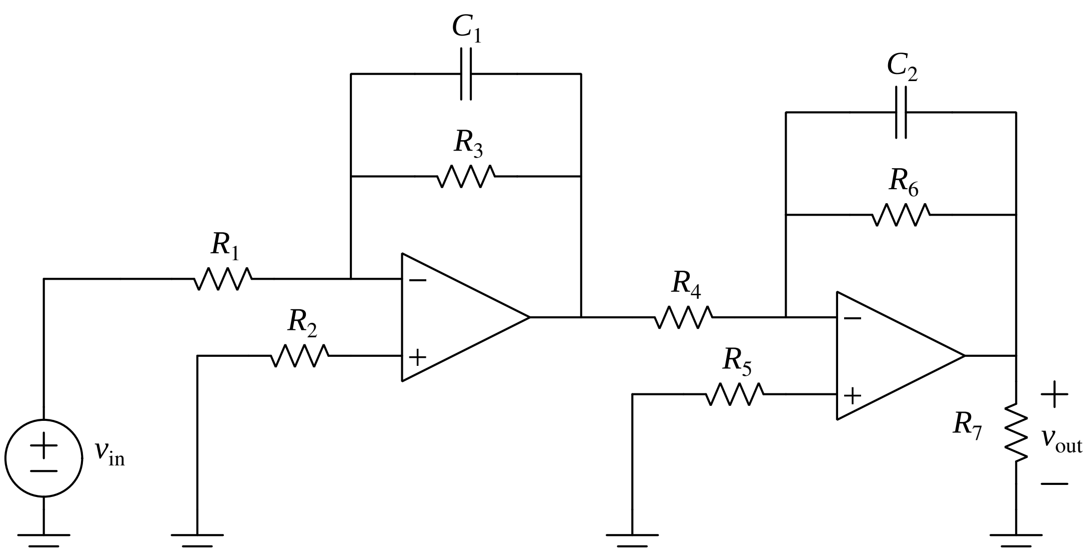
For circuits **1**, **2**, and **3** below, assuming ideal op-amps and assuming the values for all passive elements are known, find the transfer function $\mathbb{H}(s) = \frac{\mathbb{V}_{\text{out}}(s)}{\mathbb{V}_{\text{in}}(s)}$. Your final answer should be a ratio of polynomials in s.



(1)



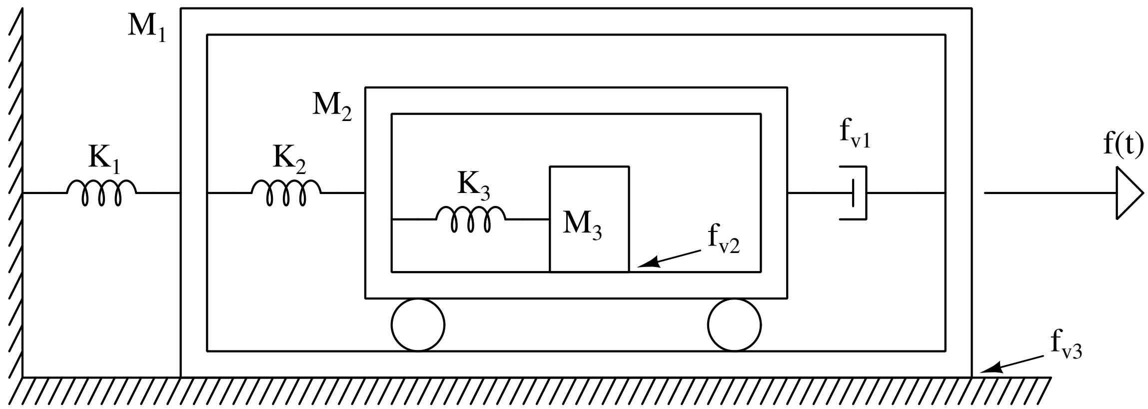
(2)



(3)

Problem IV: [10 pts.] Translation

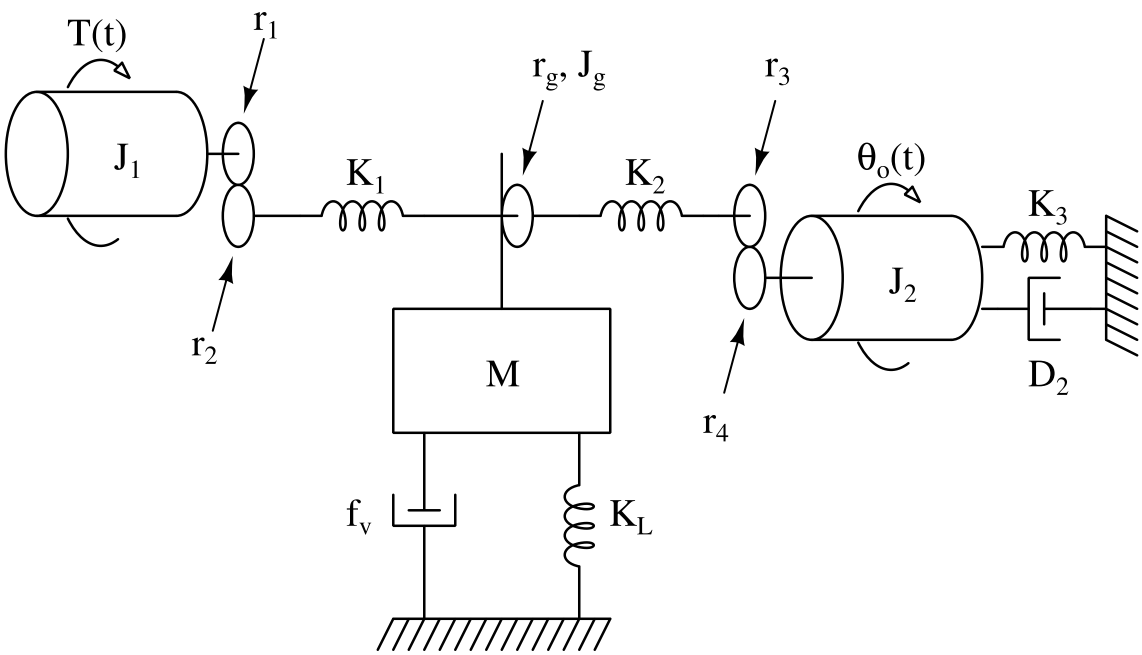
Given the following system:



where $f(t)$ is a known force applied to mass M_1 and assuming that the values of the passive elements are known, *clearly* determine a complete set of equations of motion for the system in the frequency domain. List the set of unknowns you believe your equations use and be sure they are all clearly labeled on the diagram. You do not need to arrange the equations in matrix format nor do you need to solve the equations. You may assume that the wheels are both frictionless and massless.

Problem V: [18 pts.] Rotation

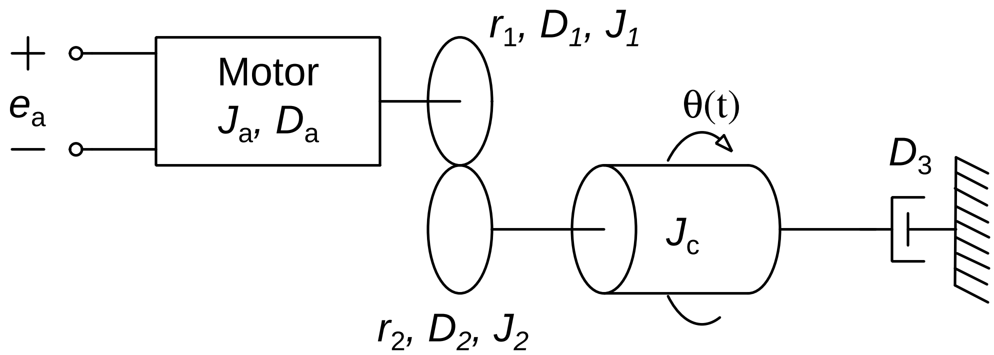
Given the following system:



where $T(t)$ is a torque applied to inertia J_1 and the output is the angle $\theta_o(t)$ through which inertia J_2 rotates, and assuming that the values of the passive elements, the torque, and the gear radii are known, *clearly* determine the equations of motion for the system in the frequency domain. List the set of unknowns you believe your equations use and be sure they are all clearly labeled on the diagram. $\mathbf{\Theta_o(s)}$ must be one of the unknowns for which your equations would solve. If you choose to introduce any additional measurements or constants, be sure to clearly define them on your diagram. You do not need to arrange the equations in matrix format nor do you need to solve the equations. Note that r_1 through r_4 represent inertialess and frictionless gears but r_g has a moment of inertia J_g.

Problem VI: [15 pts.] Electromechanics

Given the following system:



with values for the physical properties of the motor and for the passive elements of:

$J_a = 2$ kg-m^2	$J_1 = 1$ kg-m^2	$J_2 = 5$ kg-m^2	$J_c = 60$ kg-m^2
$D_a = 1$ N-m-s/rad	$D_1 = 0.5$ N-m-s/rad	$D_2 = 2$ N-m-s/rad	$D_3 = 10$ N-m-s/rad
$r_1 = 2$ cm	$r_2 = 10$ cm		

(1) Redraw the system from the perspective of the motor, including labels, and then

(2) *Clearly* determine the transfer function

$$\mathbb{H}(s) = \frac{\mathbf{\Theta}(s)}{\mathbb{E}_a(s)}$$

in terms of the element values, motor constants, and gear radii. You may assume that a positive voltage applied to the motor will result in a positive change in $\theta(t)$. Note that all the gears have their own inertia and damping. Assume that this is a 10 V DC motor. Further assume that you have performed a dynamometer test on the motor under different externally applied shaft torques and measured the following:

- With an externally applied torque of 8 N-m on the shaft, the steady state speed of the motor shaft is 4 rad/s.
- If that torque is doubled to 16 N-m, the steady state speed of the motor shaft is cut in half to 2 rad/s.