Duke University
Edmund T. Pratt, Jr. School of Engineering

Controls Spring 2025
Test II

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Name (please print):

NetID (please print):

Submitting your work for a grade implies agreement with the following: In keeping with the Community Standard, I have neither provided nor received any assistance on this test. I understand if it is later determined that I gave or received assistance, I will be brought before the Undergraduate Conduct Board and, if found responsible for academic dishonesty or academic contempt, fail the class. I also understand that I am not allowed to communicate with anyone except the instructor about any aspect of this test until the instructor announces it is allowed. I understand if it is later determined that I did communicate with another person about the test before the instructor said it was allowed, I will be brought before the Undergraduate Conduct Board and, if found responsible for academic dishonesty or academic contempt, fail the class.

Instructions

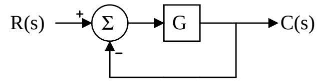
First - please turn **off** any cell phones or other annoyance-producing devices. Vibrate mode is not enough - your device needs to be in a mode where it will make no sounds during the course of the test, including the vibrate buzz or those acknowledging receipt of a text or voicemail.

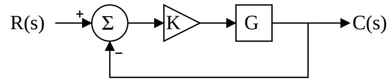
Please be sure that your name and NetID are clearly written at the top of every page. Each problem is on the back of a test page; there is space for work for that particular problem on the front of the next page. **Please be sure to put the work and answers for a given problem on the appropriate page.** Draw a box / circle / something around your final answers for each part of each problem. If you absolutely need more space for a particular problem than the extra nearly-blank page allows, put that work on its own piece of paper, clearly write your name, NetID, and the problem number (in either Arabic or Roman numerals) at the **top center** of that page, and submit any such extra pages in problem-order **after** all preprinted pages of the test. Also, on the pages given to work on a problem, write a note that says "see extra page."

You will *not* be stapling your test but instead will be turning in your test in its original folder to the box at the front of the room. Carefully stack the test pages in order (with any additional pages properly labeled and **after all the original test pages**), put them in the folder you received with the test, and bring the folder to the front of the room.

Note that there may be people taking the test after you, so you are not allowed to talk about the test - even to people outside of this class - until I send along the OK. This includes talking about the specific problem types, how long it took you, how hard you thought it was - really anything. Please maintain the integrity of this test.

In all problems, $u(t)$ is the unit step and $\delta(t)$ is the unit impulse. Also, transfer functions labeled T are generally overall transfer functions while G and H are generally subordinate forward and feedback transfer functions, respectively. Finally, the following represent the block diagrams for unity feedback systems and unity feedback systems with cascaded proportional (gain) control, respectively:





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Problem I: [20 *pts.*] Transient and Steady-State Characteristics

(1) Determine the steady-state value and the approximate settling time of the step response for a system that has an overall transfer function of:

$$T_{\mathrm{w}}(s) = \frac{24}{s^2 + 16s + 15}$$

(2) Determine the steady-state value and the approximate settling time of the step response for a unity feedback system that has a forward transfer function of:

$$G_{\mathrm{x}}(s) = \frac{24}{s^2 + 16s + 15}$$

(3) Determine the steady-state value, settling time, and %OS of the step response as well as the natural frequency, damping ratio, and damped frequency for a system that has an overall transfer function of:

$$T_{\mathrm{y}}(s) = \frac{10}{s^2 + 10s + 50}$$

$c_{\mathrm{step}}(\infty) =$ $\qquad$ $T_S =$ $\qquad$ $\%OS =$

$\omega_n =$ $\qquad$ $\zeta =$ $\qquad$ $\omega_d =$

(4) A first-order system (with no finite zeros) is found to have a step response with a steady-state value of 5 and a settling time of 3 seconds. Where is the dominant pole and what is the transfer function T for this system?

(5) An underdamped second-order system (with no finite zeros) is found to have a step response with a steady-state value of 2; the transient is found to have a maximum 15% overshoot at $t = 2$ s. Where are the dominant poles and what is the transfer function T for this system?

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Extra work for Problem I

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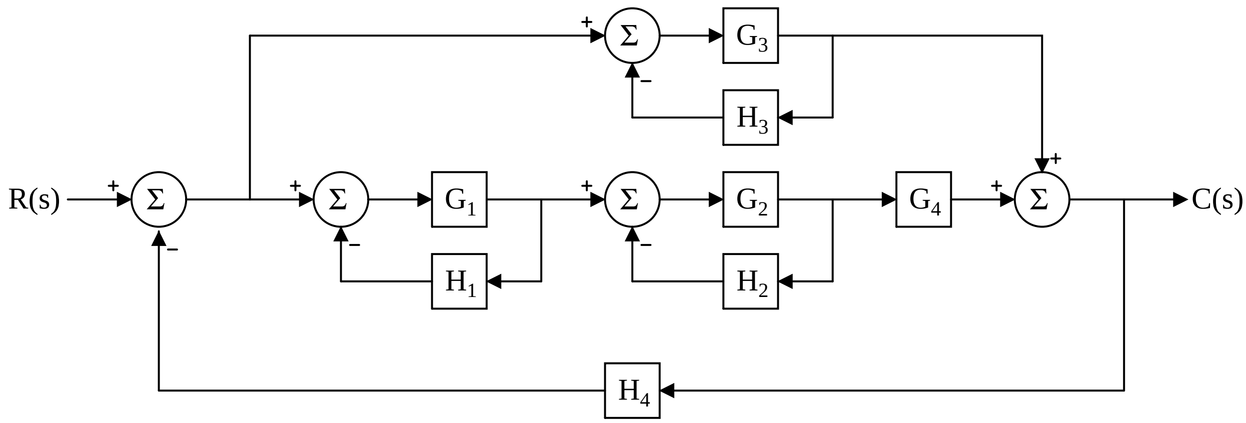
Problem II: [15 *pts.*] System Simplification

Given the system below:



(1) Clearly draw a signal flow diagram for the system. Be sure to indicate where each node is on the diagram - *especially* R and C.

(2) Clearly use Mason's Rule to determine the overall transfer function $T(s) = C(s)/R(s)$. Note that once you define components in terms of the individual transfer functions, you do *not* need to simplify nor do you need to substitute them into the final result.

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Extra work for Problem II

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Problem III: [20 *pts.*] Stability and Steady State Analysis

You must clearly build and interpret Routh arrays to support your conclusions for the following problems in order to get any credit.

(1) A system has an overall transfer function of:

$$T_\alpha(s) = \frac{1}{s^5 + 2s^4 - 8s^3 - 16s^2 - 9s - 18}$$

How many left, right, and $j\omega$ poles does the system have?

(2) A unity feedback system with gain control K has an overall forward path transfer function of:

$$G_\beta(s) = \frac{K(s+4)}{(s-8)(s+5)(s+6)} = \frac{K(s+4)}{s^3 + 3s^2 - 58s - 240}$$

(a) Determine the conditions on K for the system to be stable.

(b) Are there one or more non-trivial values of K which would cause marginal stability? If so, find them and determine the frequency of oscillation at each K where the system is non-trivially marginally stable.

(c) For values of K that keep the system stable, what is the system type from a steady-state error perspective?

(d) Given that system type, determine the value for the appropriate finite (non-zero, non-infinite) static error constant and the steady state position error as a function of K.

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Extra work for Problem III

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Problem IV: [25 *pts.*] System Analysis I

Be sure to clearly indicate which part of the problem you are answering by writing the appropriate section/subsection before your answer. For example, "(IV.2): The values of K that keep the system stable are..." A unity feedback system T has a gain controller K in cascade with a plant:

$$G(s) = \frac{s+10}{(s+1)(s+2)(s+5)} = \frac{s+10}{s^3+8s^2+17s+10}$$

(1) Is the plant G all by itself generally overdamped, critically damped, underdamped, undamped (i.e. marginally stable), or unstable? Why do you believe that?

(2) For what values of K will the overall system T be stable? Justify your answer.

(3) For values of K that keep the system stable, what system type does this represent from a steady-state error perspective?

(4) Complete the following sentence with the appropriate subscripts: "When this system is stable, the finite, non-zero static error constant is $K\_$ and the finite, non-zero position error is $e\_(\infty)$." Your first answer should be one of a, p, or v and your second should be one of parabola, ramp, or step.

(5) Determine a formula that relates the controller gain K to the finite, non-zero steady-state position error.

(6) What is the minimum possible finite, non-zero steady-state position error for this system, and what value of the controller K produces it?

(7) For one K value in the stable range, the overall system poles are at -5.04, -1.48, and -1.48 while the overall system zero is at -10. Answer the following given this information:

(a) For this gain value, what is/are the dominant pole/s?

(b) For this gain value, is the system generally overdamped, critically damped, underdamped, undamped (i.e. marginally stable), or unstable? Why do you believe that?

(8) For another K value in the stable range, the overall system poles are at -6.46 and -0.772±j4.05 while the overall system zero is at -10. Answer the following given this information:

(a) For this gain value, what is/are the dominant pole/s?

(b) For this gain value, is the system generally overdamped, critically damped, underdamped, undamped (i.e. marginally stable), or unstable? Why do you believe that?

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Extra work for Problem IV

Problem V: [20 *pts.*] System Analysis II

Be sure to clearly indicate which part of the problem you are answering by writing the appropriate section before your answer. For example, "(V.5): The system is marginally stable when..." A system with gain control has an overall transfer function T of:

$$T(s) = \frac{Ks + K}{s^4 + 4s^3 + 6s^2 + Ks + K}$$

(1) For what values of K will the overall system be stable?

(2) For values of K that keep the system stable, what system type does this represent from a steady-state error perspective?

(3) Complete the following sentence with the appropriate subscripts: "When this system is stable, the finite, non-zero static error constant is $K\_$ and the finite, non-zero position error is $e\_(\infty)$." Your first answer should be one of a, p, or v and your second should be one of parabola, ramp, or step.

(4) Determine a formula that relates the controller gain K to the finite, non-zero steady-state position error.

(5) For what value(s) of K will the overall system be non-trivially marginally stable?

(6) For the value(s) of K that makes the system non-trivially marginally stable, what frequency for the input would lead to an unbounded output?

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Extra work for Problem V

Chapter 2

Variations on Final Value Theorem (Table 2.2, p. 37)

If $T(s) = \frac{C(s)}{R(s)}$, $c(\infty) = \lim_{s\to 0} sC(s) = \lim_{s\to 0} sR(s)T(s)$
If $T(s) = \frac{C(s)}{R(s)}$ and $r(t) = u(t)$ such that $R(s) = \frac{1}{s}$, $c(\infty) = \lim_{s\to 0} sR(s)T(s) = \lim_{s\to 0} T(s)$

Chapter 4

First Order Characteristics (4.9 & 10, p. 163)

$$T_\mathrm{r} = \frac{2.2}{a} \qquad T_\mathrm{s} = \frac{4}{a}$$

Underdamped Second Order Characteristics (4.34, 38, 39, 42, and 44 , pp. 175-176)

$$T_\mathrm{p} = \frac{\pi}{\omega_n\sqrt{1-\zeta^2}} = \frac{\pi}{\omega_d} \qquad \%OS = e^{-(\zeta\pi/\sqrt{1-\zeta^2})} \times 100 \qquad \zeta = \frac{-\ln(\%OS/100)}{\sqrt{\pi^2 + \ln^2(\%OS/100)}} \qquad T_\mathrm{s} \approx \frac{4}{\zeta\omega_\mathrm{n}}$$

Underdamped Second Order Rise Time (footnote 5, p. 177)

$$\omega_\mathrm{n} T_\mathrm{r} = 1.76\zeta^3 - 0.417\zeta^2 + 1.039\zeta + 1,\ 0 < \zeta < 0.9$$

Chapter 7

Static Error Constants and Errors

$$K_\mathrm{p} = \lim_{s\to 0} G_\mathrm{eq}(s) \quad e_\mathrm{step}(\infty) = \frac{1}{1+K_\mathrm{p}} \quad K_\mathrm{v} = \lim_{s\to 0} sG_\mathrm{eq}(s) \quad e_\mathrm{ramp}(\infty) = \frac{1}{K_\mathrm{v}} \quad K_\mathrm{a} = \lim_{s\to 0} s^2 G_\mathrm{eq}(s) \quad e_\mathrm{parabola}(\infty) = \frac{1}{K_\mathrm{a}}$$

Steady State Error Via Input R and Disturbance D (7.61, 7.62, pp. 351-352)

For a unity feedback system with controller $G_1(s)$ and plant $G_2(s)$:

$$e(\infty) = \lim_{s\to 0} sE(s) = \lim_{s\to 0} \frac{s}{1+G_1(s)G_2(s)} R(s) - \lim_{s\to 0} \frac{sG_2(s)}{1+G_1(s)G_2(s)} D(s) = e_\mathrm{R}(\infty) + e_\mathrm{D}(\infty)$$
$$e_\mathrm{D,step}(\infty) = -\frac{1}{\lim_{s\to 0}\frac{1}{G_2(s)} + \lim_{s\to 0} G_1(s)}$$

General

Unity Feedback Equations; Polynomial Ratio Simplifications

$$G_\mathrm{eq} = \frac{T}{1-T} \qquad T = \frac{N_\mathrm{T}}{D_\mathrm{T}} \qquad G_\mathrm{eq} = \frac{N_\mathrm{T}}{D_\mathrm{T} - N_\mathrm{T}}$$
$$T = \frac{G_\mathrm{eq}}{1+G_\mathrm{eq}} \qquad G_\mathrm{eq} = \frac{N_\mathrm{G}}{D_\mathrm{G}} \qquad T = \frac{N_\mathrm{G}}{D_\mathrm{G} + N_\mathrm{G}}$$