Duke University
Edmund T. Pratt, Jr. School of Engineering

Controls Spring 2012

Test II

Michael R. Gustafson II

Name (please print)____

In keeping with the Community Standard, I have neither provided nor received any assistance on this test. I understand if it is later determined that I gave or received assistance, I will be brought before the Undergraduate Conduct Board and, if found responsible for academic dishonesty or academic contempt, fail the class. I also understand that I am not allowed to speak to anyone except the instructor about any aspect of this test until the instructor announces it is allowed. I understand if it is later determined that I did speak to another person about the test before the instructor said it was allowed, I will be brought before the Undergraduate Conduct Board and, if found responsible for academic dishonesty or academic contempt, fail the class.

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First Order Characteristics (4.9 & 10, p. 167)

$$T_{\mathrm{r}} = \frac{2.2}{a} \qquad\qquad T_{\mathrm{s}} = \frac{4}{a}$$

Underdamped Second Order Characteristics (4.34, 38, 39, 42, and 44, pp. 179-182)

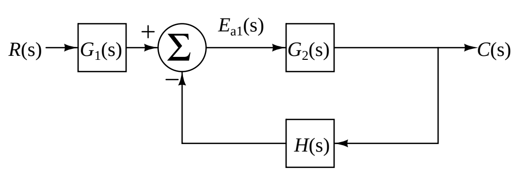
$$T_{\mathrm{p}} = \frac{\pi}{\omega_n\sqrt{1-\zeta^2}} = \frac{\pi}{\omega_d} \qquad \%OS = e^{-(\zeta\pi/\sqrt{1-\zeta^2})}\times 100 \qquad \zeta = \frac{-\ln(\%OS/100)}{\sqrt{\pi^2+\ln^2(\%OS/100)}} \qquad T_{\mathrm{s}} \approx \frac{4}{\zeta\omega_{\mathrm{n}}}$$

Unity Feedback Equations; Polynomial Ratio Simplifications

$$G_{\mathrm{eq}} = \frac{T}{1-T} \qquad T = \frac{N_{\mathrm{T}}}{D_{\mathrm{T}}} \qquad G_{\mathrm{eq}} = \frac{N_{\mathrm{T}}}{D_{\mathrm{T}}-N_{\mathrm{T}}}$$

$$T = \frac{G_{\mathrm{eq}}}{1+G_{\mathrm{eq}}} \qquad G_{\mathrm{eq}} = \frac{N_{\mathrm{G}}}{D_{\mathrm{G}}} \qquad T = \frac{N_{\mathrm{G}}}{D_{\mathrm{G}}+N_{\mathrm{G}}}$$

Actuation Error (Fig. 7.15 (a) on p. 358 and Eqn. 7/72 on p. 361)



$$e_{a1}(\infty) = \lim_{s\to 0}\frac{sR(s)G_1(s)}{1+G_2(s)H_1(s)}$$

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Problem I: [25 pts.] System Categorization and Gain Control I

Given a unity feedback system with input $r(t)$, output $c(t)$, a gain controller K, and a plant with a transfer function of:

$$G(s) = \frac{s^2 - 2s + 2}{s^2 + 4s + 3}$$

Note that a break-in/break-out point analysis reveals that

$$\frac{dK(\sigma)}{d\sigma} = 0 \text{ when } \sigma = -1.703, 1.370$$

(1) Determine the range of stability for the gain controller.

(2) Determine the gain(s) for marginal stability (if any) as well as the frequencies of oscillations for those gains (if any).

(3) Accurately sketch the root locus assuming gain control. Be sure to denote any interesting locations such as open loop poles, zeros, marginal stability frequencies, and centers of asymptotes as well as angles for the asymptotes themselves. Clearly label these points along with their values.

(4) Can the system ever be critically damped? If not, state why you believe that. If so, determine the location of the poles and the value for the gain to make the system critically damped.

(5) With respect to steady-state error, determine the system type, the value for the three static error constants (K_p, K_v, and K_a) and the steady state error assuming each of the following inputs and further assuming that the gain is set to some value K which is in the range of stability. The symbols α, β, and γ below represent constants.

(a) $r_1(t) = \alpha u(t)$
(b) $r_2(t) = \beta t u(t)$
(c) $r_3(t) = \gamma t^2 u(t)$

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Problem II: [35 pts.] System Categorization and Gain Control II

Given a system as drawn at the bottom of the cover page of this test with an input transducer G_1, an output transducer H_1, and a forward path G_2 consisting of a gain controller K in cascade with a plant such that the transfer functions for the different components are:

$$G_1(s) = 5 \qquad\qquad G_2(s) = \frac{K}{s^2} \qquad\qquad H_1(s) = \frac{s^2+4s+13}{s+1}$$

Note that for this system,

$$\frac{dK(\sigma)}{d\sigma} = 0 \text{ when } \sigma = -0.684, 0$$

(1) Determine the range of stability for the gain controller.

(2) Determine the gain(s) for marginal stability (if any) as well as the frequencies of oscillations for those gains (if any).

(3) Accurately sketch the root locus assuming gain control with $K > 0$. Be sure to denote any interesting locations such as open loop poles, zeros, marginal stability frequencies, and centers of asymptotes as well as angles for the asymptotes themselves. Clearly label these points along with their values. The stability information above should come in handy when making decisions about how to connect the dots...

(4) Can the system ever be critically damped? If not, state why you believe that. If so, determine the location of the poles and the value for the gain to make the system critically damped.

(5) With respect to steady-state error, determine the system type, the value for the three static error constants (K_p, K_v, and K_a) and the steady state error assuming each of the following inputs and further assuming that the gain is set to some value K which is in the range of stability. The symbols α, β, and γ below represent constants.

(a) $r_1(t) = \alpha u(t)$
(b) $r_2(t) = \beta t u(t)$
(c) $r_3(t) = \gamma t^2 u(t)$

(6) Determine the steady-state actuation signal e_{a1} for each of the following inputs and further assuming that the gain is set to some value K which is in the range of stability. The symbols λ, μ, and ν below represent constants.

(a) $r_1(t) = \lambda u(t)$
(b) $r_2(t) = \mu t u(t)$
(c) $r_3(t) = \nu t^2 u(t)$

(7) Given your root locus sketch, estimate the largest damping ratio this system can have using gain control. Indicate where on the root locus you expect your dominant poles to be. Give a calculation of what you expect the damping ratio might be at that location. What can you say about the gain there?

(8) Where are the zeros of the overall transfer function $T = C/R$?

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Problem III: [40 pts.] System Compensation

Given a unity feedback system with input $r(t)$, output $c(t)$, a controller $G_c(s)$, and a plant with a transfer function of:

$$G_p(s) = \frac{1}{(s)(s+2)(s+6)}$$

- Assuming the controller $G_c(s)$ is merely a gain controller K:
  - **(1)** Determine the range of stability for the gain controller.
  - **(2)** Determine the gain for marginal stability as well as the frequency of oscillations for that gain. There is one non-zero value of the gain that produces a marginally stable system.
  - **(3)** Accurately sketch the root locus assuming gain control with $K > 0$. Be sure to denote any interesting locations such as open loop poles, zeros, marginal stability frequencies, and centers of asymptotes as well as angles for the asymptotes themselves. Clearly label these points along with their values.
  - **(4)** With respect to steady-state error, determine the system type, the value for the three static error constants (K_p, K_v, and K_a) and the steady state error assuming each of the following inputs and further assuming that the gain is set to some value K which is in the range of stability. The symbols α, β, and γ below represent constants.
    - *(a)* $r_1(t) = \alpha u(t)$
    - *(b)* $r_2(t) = \beta t u(t)$
    - *(c)* $r_3(t) = \gamma t^2 u(t)$
  - **(5)** Using SISOtool, you could find that the system is underdamped with a damping ratio $\zeta = 0.7071$ when the gain K is set to 8.88. SISOtool also indicates at this point that the closed-loop poles are at:

    $$s = -6.32, -0.838 \pm j0.838$$

    Estimate the settling time for this system with this gain. How well do you think this estimate will hold up in simulation?

- Now you want to put a compensator in place of the simple gain control in order to change the transient characteristics of the system. Specifically, you want to design a compensator for $G_c(s)$ such that the estimated settling time is one-third that of the estimated settling time above but with the same damping ratio.
  - **(6)** Design a PD controller that will accomplish this. Be sure to calculate the location of any compensator poles and zeros as well as the new gain.
  - **(7)** Sketch the root locus for this compensated system.
  - **(8)** SISOtool analysis indicates that a properly designed compensator for the control requirement given will have two second-order poles as well as a first order pole at $s = -2.85$. Given all the information at your disposal, do you believe the second-order approximations apply to your system? Clearly indicate why or why not.
  - **(9)** To make the compensator passive versus active - with the possible exception of an active component to create the necessary gain - design a lead controller that has its compensator zero at $z_c = 2$. Be sure to calculate the location of any compensator poles and zeros as well as the new gain. Calculate the appropriate finite (and non-zero) static error constant and steady state error for the appropriate unit input for this system.
  - **(10)** Finally, replace the lead compensator in step **(9)** with a passive compensator that will reduce the appropriate finite (and non-zero) steady state error by a factor of four from what was found in step **(9)** without significantly changing the short-term transient characteristics found in step **(9)**. Clearly indicate the compensator poles and zeros as well as the compensator gain and the resulting finite and non-zero steady state error.