Duke University

Edmund T. Pratt, Jr. School of Engineering

Controls Spring 2012

Test I

Michael R. Gustafson II

Name (please print)____

In keeping with the Community Standard, I have neither provided nor received any assistance on this test. I understand if it is later determined that I gave or received assistance, I will be brought before the Undergraduate Conduct Board and, if found responsible for academic dishonesty or academic contempt, fail the class. I also understand that I am not allowed to speak to anyone except the instructor about any aspect of this test until the instructor announces it is allowed. I understand if it is later determined that I did speak to another person about the test before the instructor said it was allowed, I will be brought before the Undergraduate Conduct Board and, if found responsible for academic dishonesty or academic contempt, fail the class.

Signature:____

Instructions

First - please turn **off** any cell phones or other annoyance-producing devices. Vibrate mode is not enough - your device needs to be in a mode where it will make no sounds during the course of the test, including the vibrate buzz or those acknowledging receipt of a text or voicemail.

Please be sure to put each problem on its own page or pages - do *not* write answers to more than one problem on any piece of paper and do not use the back of a problem for work on a *different* problem. You will be turning in each of the problems independently. This cover page should be stapled to the front of Problem 1.

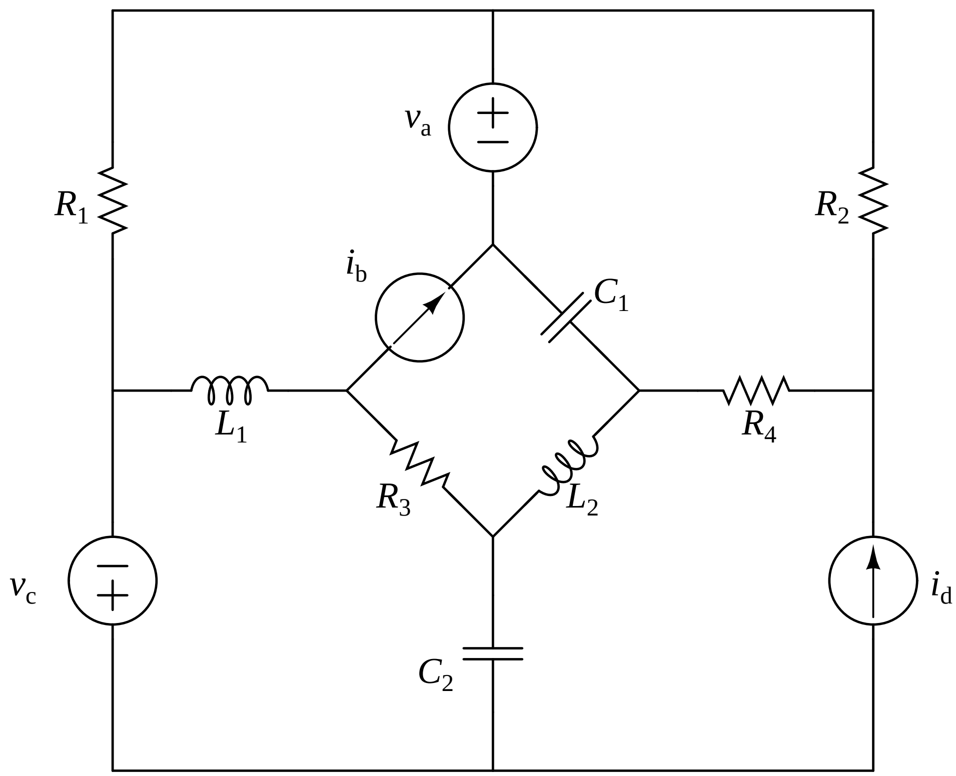
Make sure that your name *and* NET ID are *clearly* written at the top of *every* page, just in case problem parts come loose in the shuffle. Make sure that the work you are submitting for an answer is clearly marked as such. Finally, when turning in the test, individually staple all the work for each problem and place each problem's work in the appropriate folder.

Note that there may be people taking the test after you, so you are not allowed to talk about the test - even to people outside of this class - until I send along the OK. This includes talking about the specific problem types, how long it took you, how hard you thought it was - really anything. Please maintain the integrity of this test.

Name (please print):
Community Standard (print ACPUB ID):

Problem I: [15 pts.] ECE 141

Given the following electrical system:



and assuming that the values for the passive elements (L_1, L_2, R_1, R_2, R_3, R_4, C_1, and C_2), and the values for the independent sources (v_a, i_b, v_c, i_d) are known, *clearly* demonstrate the use of the Mesh Current Method in the frequency domain to label unknowns for the circuit and to determine a complete set of equations that could be used to solve for these unknowns. List the set of unknowns you believe your equations will find. You do not need to arrange the equations in matrix format.

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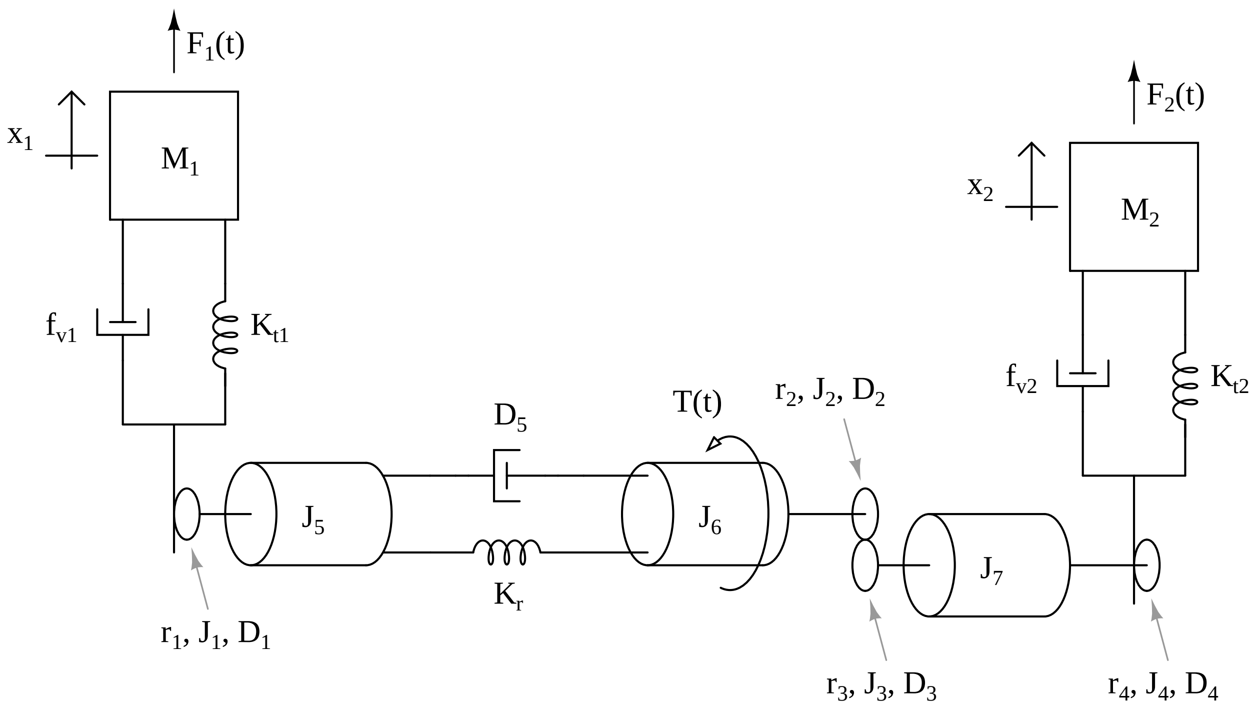
Problem II: [20 pts.] ME 125

Given the following system:



where $F_1(t)$ is a force applied to mass M_1, $F_2(t)$ is a force applied to mass M_2, $T(t)$ is a torque applied to rotational inertia J_6, x_1 is the position of mass M_1, and x_2 is the position of mass M_2,

(a) Clearly draw the system (and any labeled translations) from the perspective of the applied torque. You must include x_1 and x_2 (or what they become) as a part of your drawing. As this drawing may be quite wide, you may want to draw along the longer edge of the paper.

(b) Assuming that the values of the passive elements and of the external torque and forces are known, *clearly* determine the equations of (angular) motion for the system in the frequency domain. List the set of unknowns you believe your equations use and be sure they are all clearly labeled on the diagram. You must either explicitly include X_1 and X_2 in your equations or add two additional equations to calculate X_1 and X_2 from whatever unknown values you end up using. You do not need to arrange the equations in matrix format.

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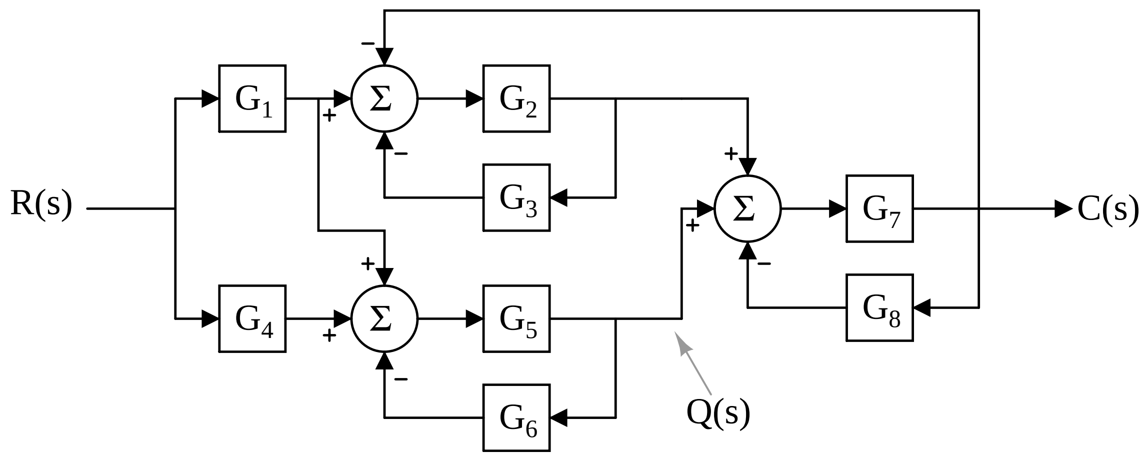
Problem III: [20 pts.] System Simplification

Given the system below:



(a) Clearly draw a signal flow diagram for the system. Be sure to indicate where each node is on the diagram - *especially* **R**, **Q**, and **C**.

(b) Use Mason's Rule to determine the overall transfer function $T(s) = C(s)/R(s)$. Note that once you define components in terms of the individual transfer functions, you do *not* need to simplify nor do you need to substitute them into the final result.

(c) Use Mason's Rule to determine the transfer function $T(s) = Q(s)/R(s)$. Note that once you define components in terms of the individual transfer functions, you do *not* need to simplify nor do you need to substitute them into the final result. If you re-use any components defined above, simply state that.

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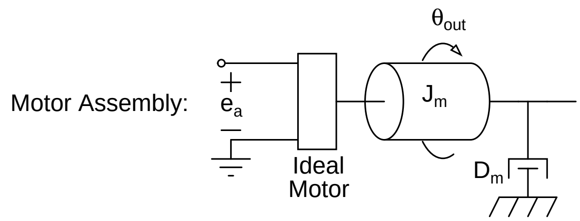
Problem IV: [25 pts.] An Engineer Feels The Need[1]

An engineer is planning to develop a speed controller for a DC motor and starts the design process by putting the motor in the following motor assembly:

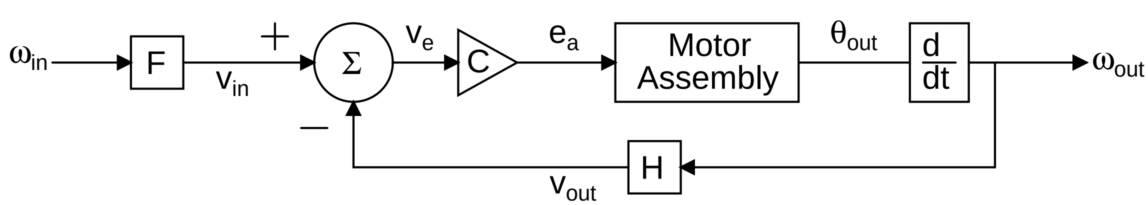


The engineer is modeling the motor as ideal because the load inertia and damping will be much, much larger than the internal armature inertia and damping.

(a) Write a symbolic equation for the transfer function of the motor assembly, $\mathcal{G}_M(s) = \frac{\Theta_{\text{out}}(s)}{E_{\text{a}}(s)}$ assuming you will later be able to determine the values of the motor's armature resistance, back emf constant, and torque constant.

(b) The engineer tests the motor by applying a constant input voltage of 10 V. The engineer determines that the no load speed is 5 rad/s and that the motor shaft stops completely when the engineer applies a torque of 40 N·m. Furthermore, the engineer establishes that the total rotational inertia attached to the motor, J_{m}, is 100 kg·m^2 but is unable to directly determine the damping. Another experiment shows, however, that applying a constant voltage to the motor causes it to stay within 2% of its terminal angular speed after 10 seconds. Clearly show how the engineer can determine the damping in the system from this information, then find the damping.

With the motor assembly now established, the engineer places it in a feedback system designed to control the speed of the motor:



The desired input angular speed is sent to the system as ω_{in}. The transfer function $\mathcal{F}$ converts the measurement in rad/s to a voltage. This voltage is then connected to the positive input of a difference block. The actual output angular speed is measured by sending the output angular position through a differentiation block. This angular speed is converted to a voltage using a transfer function $\mathcal{H}$, and the voltage from the output of that transfer function is connected to the negative input of a difference block. The voltage difference coming out of that block is then sent through a compensator $\mathcal{C}$ to produce the voltage applied to the motor assembly. The way the system is connected, a positive value of the motor voltage e_{a} will lead to a positive change in the output angle θ_{out}.

(c) Using the *symbolic* transfer function from part *(a)* above, and assuming the transfer functions $\mathcal{F}$, $\mathcal{H}$, and $\mathcal{C}$ are just constants F, H, and C, respectively, find the overall transfer function $\mathcal{T}(s) = \frac{\omega_{\text{out}}(s)}{\omega_{\text{in}}(s)}$

(d) Assuming C=1 and using the *numerical* transfer function for the motor assembly from part *(b)* above, determine the numerical value(s) of F and/or H such that the system stays within 2% of its terminal speed within 2 seconds. Explain why you believe this is correct.

[1]The Need For Speed.

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Problem V: [20 pts.] Analytical Tools That Haven't Come Up Yet

(1) A controlled system is found to have an overall transfer function:

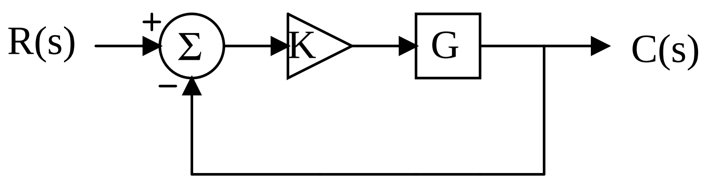
$$T_1(s) = \frac{C(s)}{R(s)} = \frac{1}{s^6 - s^5 - 3s^4 + 3s^3 - 4s^2 + 4s}$$

Clearly use a Routh Table to determine the general location of all the poles of the system. Is the system stable? If not, is the system marginally stable?

(2) A *totally different* controlled system is made up of a gain controller K cascaded with a plant

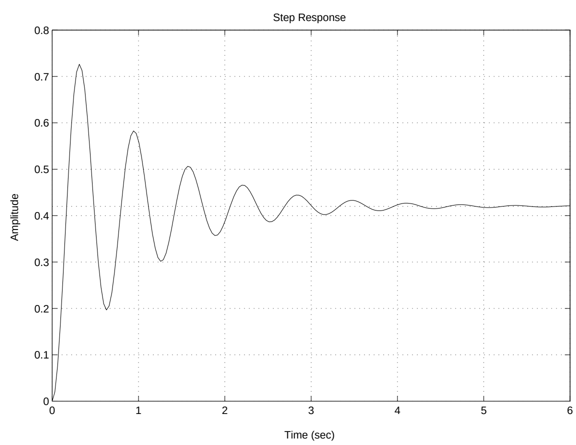
$$G(s) = \frac{1}{(s+3)(s+2)(s-1)}$$

This system is put in the forward path of a unity feedback system, which is to say:



Clearly use a Routh Table to determine the range of stability for the gain K in the system. Also determine any gain(s) that will cause marginal stability and calculate the frequency of oscillation for the system with such gain(s).

(3) A *completely unrelated* system - which can be modeled using a second order transfer function, is found to have a step response of:



(a) Determine a numerical estimate of the transfer function, and explain how you obtained any numbers in your estimate.

(b) If you multiplied this system by $\frac{3}{s+3}$, are the second-order approximations for the transient characteristics likely be valid? Explain why or why not.

(c) If you multiplied this system by $\frac{\frac{s}{3.1}+1}{\frac{s}{3}+1}$, are the second-order approximations for the transient characteristics likely be valid? Explain why or why not.