

Controls Test 8 Spring 2012

Note Title

$$J: G(s) = \frac{s^2 - 2s + 2}{s^2 + 4s + 3} = \frac{(s-1)^2 + (1)^2}{(s+1)(s+3)}$$

$$H(s) = 1$$

$$T = \frac{KG}{1+KGH} = \frac{K(s^2 - 2s + 2)}{s^2 + 4s + 3 + Ks^2 - 2sK + 2K}$$

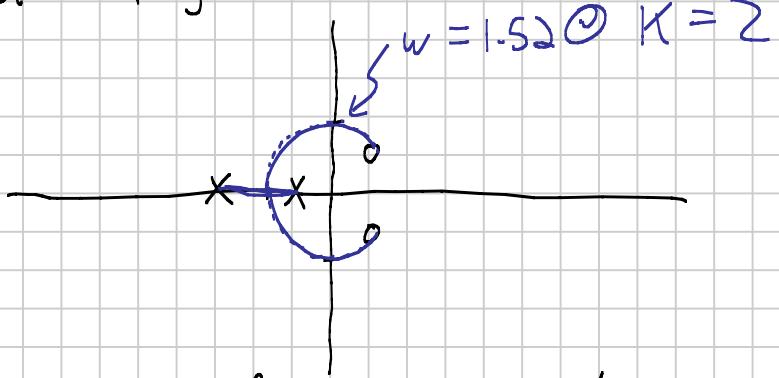
$$\begin{array}{l} 1) \quad \begin{array}{ccc|c} s^2 & 1+K & 3+2K & K > -1 \\ s^1 & 4-2K & & K < 2 \\ s^0 & 3+2K & & K > -\frac{3}{2} \end{array} \\ \qquad \qquad \qquad \begin{array}{l} \text{so} \\ -1 < K < 2 \end{array} \end{array}$$

2) $K = -1$ does not produce row of 0; not marginal

$K=2$ means s^1 is all zero, look at s^2

$$3s^2 + 7 = 0 \quad s = \pm j\sqrt{\frac{7}{3}} \quad w = \sqrt{\frac{7}{3}} = 1.52$$

3) open-loop poles at $-1, -3$
open-loop zeros at $-1 \pm j$
no asymptotes



4) yes; when $\sigma = -1.703$ there is a break.

$$K = -\frac{1}{GH} = 0.1097$$

5) Type 0 from G, so

$$K_p = \lim_{s \rightarrow 0} KG = \frac{2K}{3} \quad e_{ss} = \frac{\alpha}{1+K_p} - \frac{3\alpha}{3+2K}$$

$$K_V = 0$$

$$K_a = 0$$

$$e_{ss} = \infty$$

$$\bar{e}_{ss} = \infty$$

$$II \quad G_1 = 5 \quad G_2 = \frac{K}{s^2} \quad H_1 = \frac{s^2 + 4s + 13}{s+1} = \frac{(s+2)^2 + (3)^2}{(s+1)}$$

$$(1) \quad T = \frac{G_1 G_2}{1 + G_2 H_1} = \frac{\frac{5K}{s^2}}{1 + \frac{K}{s^2} \frac{s^2 + 4s + 13}{s+1}} = \frac{5K(s+1)}{s^3 + s^2 + Ks^2 + 4Ks + 13K}$$

s^3	1	$4K$	$K > -1$ $K(4K-9) > 0$ $\begin{cases} K > 6 \\ K > 1/4 \end{cases}$
s^2	$1+K$	$13K$	
s^1	$\frac{4K^2 - 9K}{1+K}$		
s^0	$13K$	$K > 0$	

$K > 1/4$

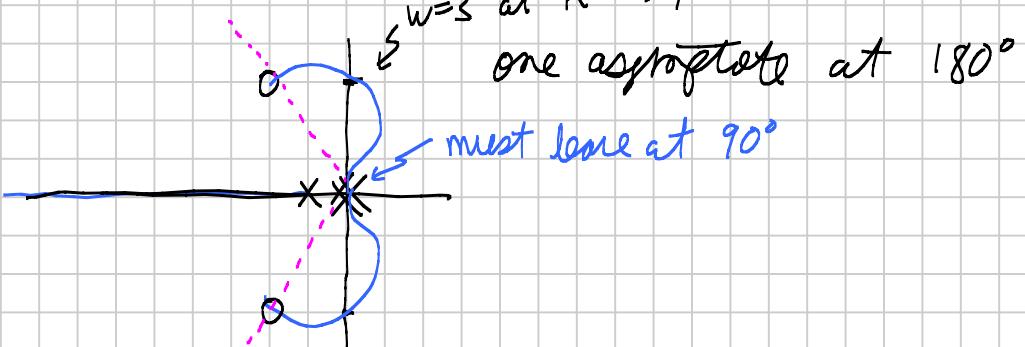
(2) $K = 1/4$ makes s^1 all zeros; put into $s^2 +$

$$\frac{13}{4}s^2 + \frac{117}{4} = 0 \quad s = \pm j\sqrt{3} \quad w = \underline{3}$$

(3) open loop poles at $0, 0, -1$

open loop zeros at $-2 \pm j\sqrt{3}$

$w = 3$ at $K = 1/4$



(4) only double pole when $K=0$, no no critical point.

(5) G_{eff} for unity feedback is

$$\frac{T}{T-T} = \frac{SK(s+1)}{s^3 + s^2 + Ks^2 + 4Ks + 13K - SKs - SK} = \frac{SK(s+1)}{s^3 + (1+K)s^2 - KS + 8K}$$

if $K < 0$, Type 0 so $K_p = 5/8$ $e_{ss} = \alpha \frac{1}{1+K_p} = \underline{\alpha \frac{8}{13}}$

$$K_v = 0$$

$$K_a = 0$$

(6)

$$e_{a1}(\infty) = \lim_{s \rightarrow 0} \frac{s R G_1}{1 + G_2 H_1}$$

$$\frac{(s) R (s)}{1 + \frac{K(s^2 + 4s + 13)}{(s^2)(s+1)}} = \frac{5(s^3)(s+1) R}{s^3 + s^2 + K(s^2 + 4s + 13)}$$

if $R = \frac{1}{s}$ or $\frac{1}{s^2}$, $e_{a1}(\infty) = 0$

$$\text{if } R = \frac{1}{s^3}, e_{a1}(\infty) = \lim_{s \rightarrow 0} \frac{5(s+1)}{s^3 + s^2 + K(s^2 + 4s + 13)} = \frac{5}{13K}$$

so

$$1 u(t) \rightarrow 0$$

$$utu(t) \rightarrow 0$$

$$\sqrt{t^2} u(t) \rightarrow \frac{2\sqrt{5}}{13K} = \frac{10}{13} \sqrt{\frac{K}{5}}$$

(7) "largest damping" means smallest

θ

as $K \rightarrow \infty$ poles go to $-2 \pm j3$

$$\frac{\zeta}{\sqrt{1-\zeta^2}} = \frac{2}{3} \quad 3\zeta = 2\sqrt{1-\zeta^2}$$

$$9\zeta^2 = 4 - 4\zeta^2 \quad 13\zeta^2 = 4$$

$$\zeta = 0.555$$

(8) zeros of T are zeros of G_2 (none) and

poles of H_1 (-1) -1

$$\text{III} \quad G_p = \frac{1}{(s)(s+2)(s+6)} = \frac{1}{s^3 + 8s^2 + 12s}$$

$$(1) \quad G_c = K \quad T = \frac{KG_p}{1+KG_p} = \frac{K}{s^3 + 8s^2 + 12s + K}$$

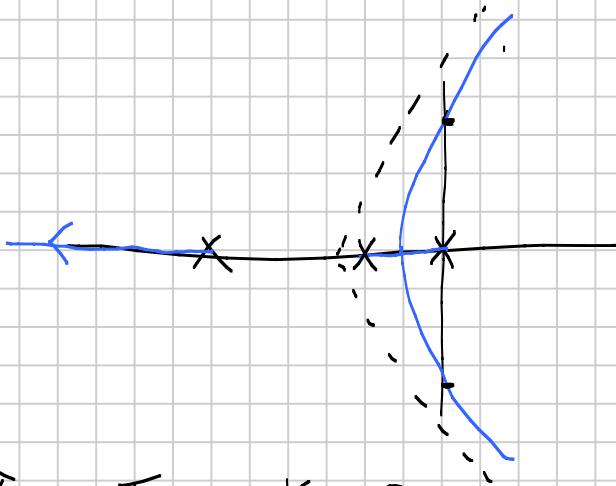
$$\begin{array}{c|cc} s^3 & 1 & 12 \\ s^2 & 8 & K \end{array}$$

$$\begin{array}{cc} s^1 & \frac{96-K}{8} \\ s^0 & K \end{array} \quad K < 96 \quad K > 0$$

$$0 < K < 96$$

$$(2) \quad K = 96 \quad \text{means} \quad 8s^2 + 96 = 0 \quad s = j\sqrt{12} \quad w = \sqrt{12} = 3.464 \text{ rad/s}$$

(3)



$$\Gamma_a = \frac{0-2-\theta}{j} = -\frac{8}{j}$$

$$\Theta_a = 60^\circ + 120^\circ$$

(4) Type I

$$K_p = \infty \quad e_{ss} = 0$$

$$K_v = \frac{K}{j\omega_n} \quad e_{ss} = \frac{\beta}{K_v} = \frac{12\beta}{K_v}$$

$$K_a = 0 \quad e_{ss} = \infty$$

$$(5) \quad T_s = \frac{4}{\zeta \omega_n} = \frac{4}{0.838} = 4.773 \text{ sec.}$$

Since there are no zeros and the real part of the 3rd pole is more than $5 \times \zeta \omega_n$, 2nd order approx should apply.

(6) To get T_s to $1/3$, $\zeta \omega_n = 3 \times 0.838 = 2.514$

$\zeta = 0.7071$ is a 45° line, so $\omega_n \sqrt{1-\zeta^2} = 2.514$ as well

$$s_{\text{design}} = -2.514 \pm j2.514$$

$$G_c G_p = \frac{K (s+2)}{(s)(s+2)(s+6)}$$

$$\angle G_c G_p = \angle K + \angle s+2 - \angle s - \angle s+2 - \angle s+6 = 180(2k+1)$$

$$0^\circ + \angle s+2 - 135^\circ - 101.55^\circ - 35 - 80^\circ = 180(2k+1)$$

$$\angle s+2 = 272.35 + 180(2k+1) = 92.35^\circ$$

pitch $k=-1$

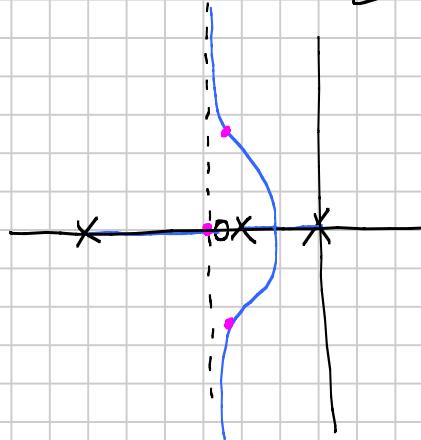
$$\frac{2 \cdot 514}{z_c - 2 \cdot 514} = \tan 92.35^\circ = -21.37$$

$$z_c = \frac{2 \cdot 514}{-21.37} + 2 \cdot 514 = 2.410$$

$$K = \frac{-1}{G_c G_p} = \frac{(s)(s+2)(s+6)}{(s+2.410)} \Big|_{s=-2 \cdot 514 + j \cdot 2 \cdot 514} = 15.6$$

$$\underline{\text{so}} \quad G_c = 15.6 (s + 2.410)$$

7)



$$r_a = \frac{0 - 2 - 6 + 2.410}{2} = -2.795$$

8) Extra pole at -2.85 may be cancelled by zero at 2.410
but seems a bit far.

$$9) \quad G_c G_p = \frac{K (s+2)}{(s)(s+2)(s+6)(s+p)} = \frac{K}{(s)(s+6)(s+p)}$$

$$\angle G_c G_p = \angle K - \angle s - \angle s+6 - \angle s+p = 180(2k+1)$$

$$0 - 135^\circ - 35 - 80^\circ - \angle s+p = 180(2k+1)$$

$$\angle s+p = -170 - 8 - 180(2k+1) = 9.2^\circ$$

pitch $k=-1$

$$p_c = \frac{2 \cdot 514}{\tan(9.2^\circ)} + 2 \cdot 514 = 18.04 \quad K = \frac{-1}{G_p} = \frac{(s)(s+6)(s+18.04)}{1} = 240$$

$$K_V = \frac{K}{(2)(6)(18.04)} = 2022$$

$$ess = 0.0451$$

(10) To get factor of 4 improvement, with $K_{V,\text{new}} = 4K_{V,\text{old}}$

\Rightarrow use lag. $\frac{s+2c}{s+p_c}$ and $z_c = 4 p_c$

pick $z_c = p_c$ so that z_c is not too close to real part of 2nd order poles (i.e. 2.514) but not too close to origin either. Try $-1 + 0.025$

so

$$KG_cG_p = \frac{K(s+2)(s+0.1)}{(s)(s+2)(s+6)(s+18.04)(s+0.025)}$$

$$K = -\left. \frac{1}{GH} \right|_{s=s_{\text{ref}}} = \frac{(s)(s+6)(s+18.04)(s+0.025)}{(s+0.1)} = 244$$

$$K_V = \frac{(244)(2)(-1)}{(-2)(6)(18.04)(-0.025)} = 9.01$$

$$ess = \frac{1}{K_V} = 0.111$$