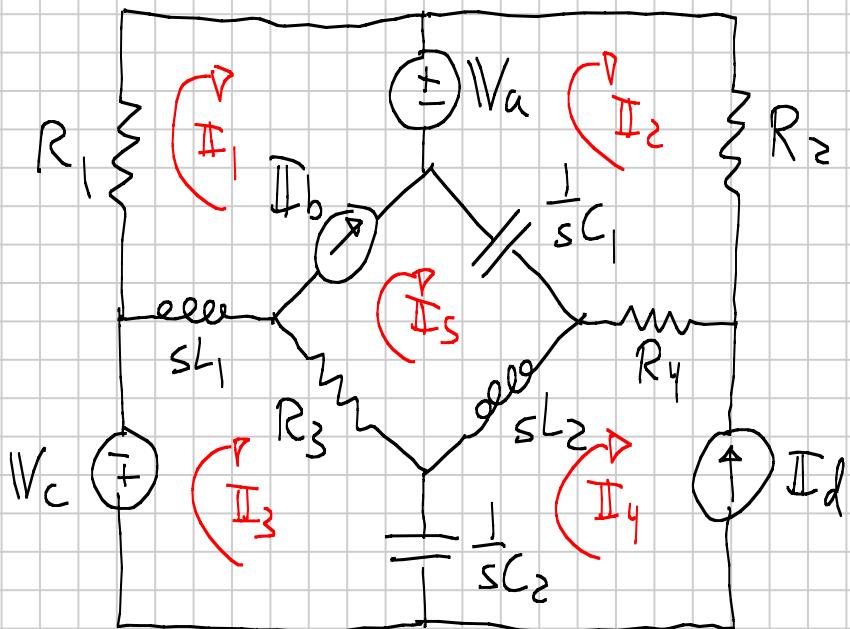


Controls Test 1 Spring 2012

Note Title

I)



5 mesh - 2 i source = 3 KVL + 2 AUX equations
 Avoid current source for \uparrow

$$\text{KVL, } sL_1: R_1 I_1 + V_a + \frac{1}{sC_1} (I_5 - I_2) + sL_2 (I_5 - I_4) + R_3 (I_5 - I_3) + sL_1 (I_1 - I_3) = 0$$

$$\text{KVL, } l_2: -V_a + R_2 (I_2) + R_4 (I_2 - I_4) + \frac{1}{sC_1} (I_2 - I_5) = 0$$

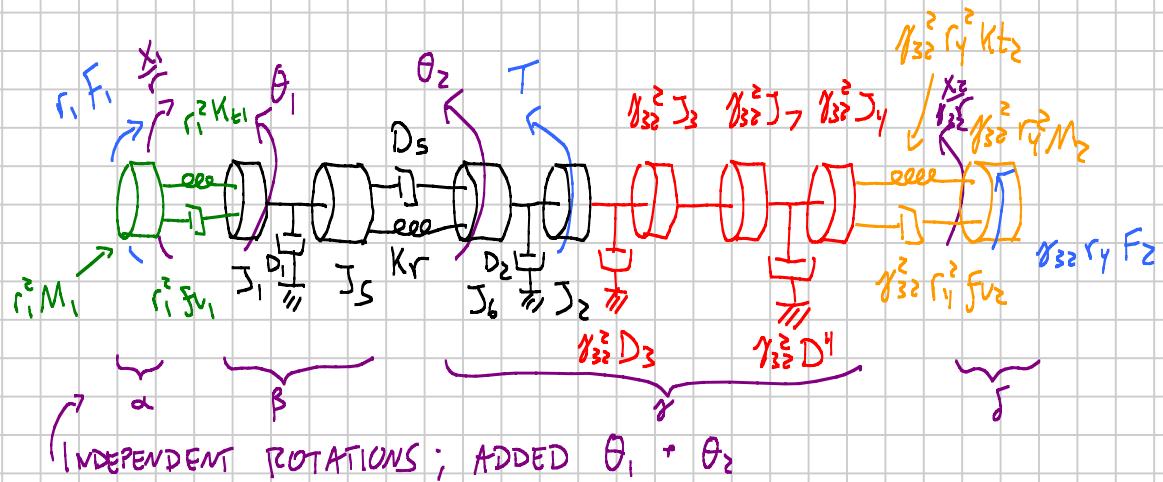
$$\text{KVL, } l_3: V_c + sL_1 (I_3 - I_1) + R_3 (I_3 - I_5) + \frac{1}{sC_2} (I_3 - I_4) = 0$$

$$\text{AUX 1: } I_b = I_5 - I_1$$

$$\text{AUX 2: } I_d = -I_4$$

II) Colors represent similar reflections

a)



$$J_\alpha = r_i^2 M_1 \quad J_\beta = J_1 + J_S \quad J_\gamma = J_2 + J_3 + J_4 \quad J_\delta = r_{32}^2 r_y^2 M_2$$

b)

$$\alpha: (J_\alpha s^2 + r_i^2 f_{v1}s + r_i^2 K_{t1})(-\frac{x_1}{r}) - (r_i^2 f_{v1}s + r_i^2 K_{t1})(\theta_1) = -r_i F_i$$

$$\beta: -(r_i^2 f_{v1}s + r_i^2 K_{t1})(-\frac{x_1}{r}) + (J_\beta s^2 + (r_i^2 f_{v1} + D_1 + D_S)s + (r_i^2 K_{t1} + Kr))\theta_1 - (D_S s + Kr)\theta_2 = 0$$

$$\gamma: -(D_S s + Kr)\theta_1 + (J_\gamma s^2 + (D_S + D_2 + r_{32}^2 D_3 + r_{32}^2 D_4 + r_{32}^2 r_y^2 f_{v2})s + (Kr + r_{32}^2 r_y^2 K_{t2}))\theta_2 - (r_{32}^2 r_y^2 f_{v2}s + r_{32}^2 r_y^2 K_{t2})\left(\frac{x_2}{r_{32} r_y}\right) = T$$

$$\delta: -(r_{32}^2 r_y^2 f_{v2}s + r_{32}^2 r_y^2 K_{t2})\theta_2 + (J_\delta s^2 + r_{32}^2 r_y^2 f_{v2}s + r_{32}^2 r_y^2 K_{t2})\left(\frac{x_2}{r_{32} r_y}\right)$$

$$r_{32} r_y F_2$$

Could define: $D_{\alpha 0} = 0 \quad D_{\beta 0} = D_1 \quad D_{\gamma 0} = D_2 + r_{32}^2 D_3 + r_{32}^2 D_4 \quad D_{\delta 0} = 0$

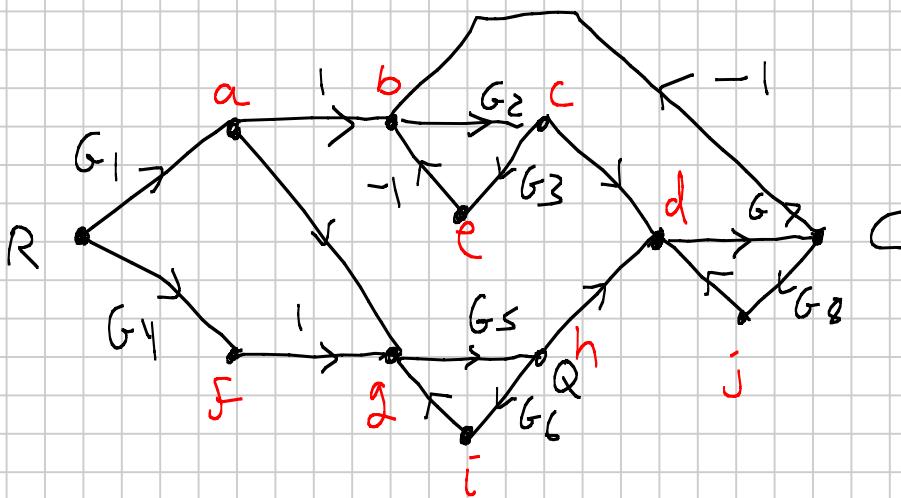
$$D_{\alpha \beta} = r_i^2 f_{v1} \quad D_{\beta \gamma} = D_S \quad D_{\gamma \delta} = r_{32}^2 r_y^2 f_{v2}$$

$$K_{\alpha 0} = 0 \quad K_{\beta 0} = 0 \quad K_{\gamma 0} = 0 \quad K_{\delta 0} = 0 \quad -$$

$$K_{\alpha \beta} = r_i^2 K_{t1} \quad K_{\beta \gamma} = Kr \quad K_{\gamma \delta} = r_{32}^2 r_y^2 K_{t2}$$

$$\begin{bmatrix} (J_\alpha s^2 + (D_{\alpha 0} + D_{\alpha \beta})s +) & -D_{\alpha \beta}s - K_{\alpha \beta} & 0 & 0 \\ -D_{\beta \gamma}s - K_{\alpha \beta} & (J_\beta s^2 + (D_{\beta 0} + D_{\beta \gamma} + D_{\beta \delta})s +) & -D_{\beta \gamma}s - K_{\beta \gamma} & 0 \\ 0 & -D_{\beta \gamma}s - K_{\beta \gamma} & (J_\gamma s^2 + (D_{\gamma 0} + D_{\gamma \delta} + D_{\gamma \delta})s +) & -D_{\gamma \delta}s - K_{\gamma \delta} \\ 0 & 0 & -D_{\gamma \delta}s - K_{\gamma \delta} & (J_\delta s^2 + (D_{\delta 0} + D_{\delta \gamma})s +) \end{bmatrix} \begin{bmatrix} -\frac{x_1}{r} \\ \theta_1 \\ \theta_2 \\ -\frac{x_2}{r_{32} r_y} \end{bmatrix} = \begin{bmatrix} -r_i F_i \\ 0 \\ T \\ r_{32} r_y F_2 \end{bmatrix}$$

III)



b)

$$T_1 : R a b c d C \quad G_1 G_2 G_7$$

$$L_1 : b c e b \quad -G_2 G_3$$

$$T_2 : R a g h d C \quad G_1 G_5 G_7$$

$$L_2 : g h i g \quad -G_5 G_6$$

$$T_3 : R f g h d C \quad G_4 G_5 G_7$$

$$L_3 : d C j d \quad -G_7 G_8$$

$$L_4 : b c d C b \quad -G_2 G_7$$

$$\Delta = 1 - L_1 - L_2 - L_3 - L_4 + L_1 L_2 + L_1 L_3 + L_2 L_3 + L_2 L_4 - L_1 L_2 L_3$$

$$\Delta_1 = 1 - L_2$$

$$\frac{C}{R} = \frac{T_1 \Delta_1 + T_2 \Delta_2 + T_3 \Delta_3}{\Delta}$$

$$\Delta_2 = 1 - L_1$$

c) L and Δ as above

$$T_1 : R a g Q \quad G_1 G_5$$

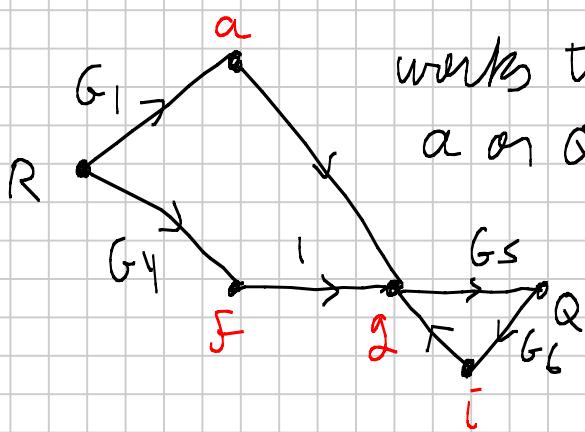
$$T_2 : R f g Q \quad G_4 G_5$$

$$\Delta_1 = 1 - L_1 - L_3 - L_4 + L_1 L_3$$

$$\frac{Q}{R} = \frac{T_1 \Delta_1 + T_2 \Delta_2}{\Delta}$$

$$\Delta_2 = 1 - L_1 - L_3 - L_4 + L_1 L_3$$

Note:



works too as nothing forward of a or Q flows back!

$$T_1 : R a g Q \quad G_1 G_5 \quad L_1 g Q i g G_5 G_6$$

$$T_2 : R f g Q \quad G_4 G_5$$

$$\Delta = 1 - L_1 \quad \frac{Q}{R} = \frac{T_1 \Delta_1 + T_2 \Delta_2}{\Delta}$$

$$\text{IV) } a) \frac{G_m - H}{E_a} = \frac{\frac{1}{J_m} \frac{K_t}{R_a}}{s(S + \frac{1}{J_m}(D_m + \frac{K_t K_b}{R_a}))}$$

b) if $e_a = 10 \text{ V}$ $\omega_{\text{no-load}} = 5 \text{ rad/s}$ $T_{\text{stall}} = 40 \text{ N}\cdot\text{m}$

$$\frac{K_t}{R_a} = \frac{T_{\text{stall}}}{e_a} = 4 \quad K_b = \frac{e_a}{\omega_{\text{no-load}}} = 2$$

$$\underline{G_m} = \frac{H}{E_a} = \frac{\frac{1}{100} \times 4}{s(S + \frac{1}{100}(D_m + 8))} = \frac{0.04}{s(S + (\frac{D_m}{100} + 0.08))}$$

$$\text{Speed is } \frac{d\theta}{dt} \text{ so } \underline{\frac{d\theta}{E_a}} = s G_m = \frac{0.04}{s + (\frac{D_m}{100} + 0.08)}$$

$$1^{\text{st}} - \text{order system} \quad T_s = 10 = \frac{4}{a} = \frac{4}{\frac{D_m}{100} + 0.08}$$

$$D_m = (.4 - .08) \times 100 = 32 \text{ N}\cdot\text{m}\cdot\text{s}/\text{rad} = 32 \text{ kg}\cdot\text{m}^2/\text{rad}$$

c) Forward path is $C G_m s$

$$\text{Feedback is } H \quad \underline{T} = F \left(\frac{C G_s}{1 + H C G_s} \right)$$

Input x-ducer is F

$$d) \quad \underline{T} = F \left(\frac{\frac{0.04}{s + 0.4}}{1 + \frac{0.04 H}{s + 0.4}} \right) = \frac{0.04 F}{s + 0.4 + 0.04 H}$$

if $C=1$,

$$T_s = 2 = \frac{4}{a} = \frac{4}{0.4 + 0.04 H} \quad H = \frac{2 - 0.4}{0.04} = 40$$

Changing F does not change transient
(other than steady state value)

IV) (I)

$\curvearrowleft R$	$s^6 + 1$	-3	-4	0
$\overline{s^5} - \sim 1$	3	4		
NOTR	$s^4 -$	$\begin{vmatrix} 1 & -3 \\ -1 & 3 \end{vmatrix} = 0$	$\begin{vmatrix} 1 & -4 \\ -1 & 4 \end{vmatrix} = 0$	0
R	$s^3 +$	$\begin{vmatrix} -(-1) & -5 \\ -1 & 3 \end{vmatrix} = 6$	$\begin{vmatrix} -(-1) & 9 \\ -1 & 4 \end{vmatrix} = 16$	$\times \frac{5}{2} \text{ to simplify}$
NOTR	$s^2 +$	$\begin{vmatrix} -5 & 9 \\ -(-5) & 3 \end{vmatrix} = 67$	$4/12$	$\times 3 \text{ to simplify}$
NOTR	$s^1 +$	$\begin{vmatrix} 3 & 8 \\ -3 & 3 \end{vmatrix} = 67$		
NOTR	$s^0 +$	$\begin{vmatrix} 3 & 8 \\ 67 & 12 \end{vmatrix} = 500$		
		12	67	

From s^5 row. $-s^5 + 3s^3 + 4s^1$ is a factor

$$\frac{d}{ds} = -5s^4 + 9s^2 + 4$$

Regular: IR OL

Special: IR, so IL 5 total, so $\underline{3jw}$

Total: 2R 1L $\underline{3jw}$

NOT STABLE (AT LEAST ONE R)

NOTE: MARGINAL ONLY POSSIBLE IF

* NO RIGHT AND

* jw POLES OF MULTIPLICITY 1

$$(2) G = \frac{K}{s^3 + 4s^2 + s - 6}$$

$$T = \frac{N_G}{D_G + N_G} = \frac{K}{s^3 + 4s^2 + s - 6 + K}$$

s^3	1	1	
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s^2	4	$K-6$	
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s^1	$\begin{vmatrix} 1 & 1 \\ 4 & K-6 \end{vmatrix} = 10-K$	$\frac{10-K}{4} > 0$	$K < 10$
s^0	$K-6$	$K-6 > 0$	$K > 6$

s^0 $6 < K < 10$

Marginal at $K=6$: $\left(\frac{10-(6)}{4} \right) s = 0 \quad s=0 \quad w=0 \text{ rad/s}$

$$K=10 : 4s^2 + (10-6) = 0$$

$$4s^2 + 4 = 0 \quad s = \pm j \quad w = 1 \text{ rad/s}$$

NEITHER ARE TRIVIAL SINCE $K \neq 0$..

(3) From drawing, steady state ≈ 0.42 (K)

$$T_p \approx 0.3 \text{ sec}$$

$$\%OS \approx \frac{0.72 - 0.42}{0.42} \approx 71.48\%$$

$$\zeta = \frac{-\ln(\%OS)}{\sqrt{T_p^2 + \ln^2(\%OS)}} = 0.1065 \quad w_n = \frac{\pi}{T_p \sqrt{1-\zeta^2}} \approx 10.5$$

$$\frac{\zeta w_n^2}{s^2 + 2\zeta w_n s + w_n^2} \rightarrow \frac{46.305}{s^2 + 2.0243s + 110.9}$$

< actual used was $K=0.42, \zeta=0.1, w_n=10$ for

$$\frac{42}{s^2 + 2s + 100}$$

b) Real part of roots at ≈ -1 , so
 -3 is not $5x$ further; approx not likely valid

c) -3 and $-3-1$ are within $\approx 3\%$; close enough for pole/zero cancellation; approx valid