Duke University
Edmund T. Pratt, Jr. School of Engineering

Controls Spring 2011

Test I

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Name (please print)____

In keeping with the Community Standard, I have neither provided nor received any assistance on this test. I understand if it is later determined that I gave or received assistance, I will be brought before the Undergraduate Conduct Board and, if found responsible for academic dishonesty or academic contempt, fail the class. I also understand that I am not allowed to speak to anyone except the instructor about any aspect of this test until the instructor announces it is allowed. I understand if it is later determined that I did speak to another person about the test before the instructor said it was allowed, I will be brought before the Undergraduate Conduct Board and, if found responsible for academic dishonesty or academic contempt, fail the class.

Signature:____

Instructions

First - please turn **off** any cell phones or other annoyance-producing devices. Vibrate mode is not enough - your device needs to be in a mode where it will make no sounds during the course of the test, including the vibrate buzz or those acknowledging receipt of a text or voicemail.

Please be sure to put each problem on its own page or pages - do *not* write answers to more than one problem on any piece of paper and do not use the back of a problem for work on a *different* problem. You will be turning in each of the problems independently. This cover page should be stapled to the front of Problem 1.

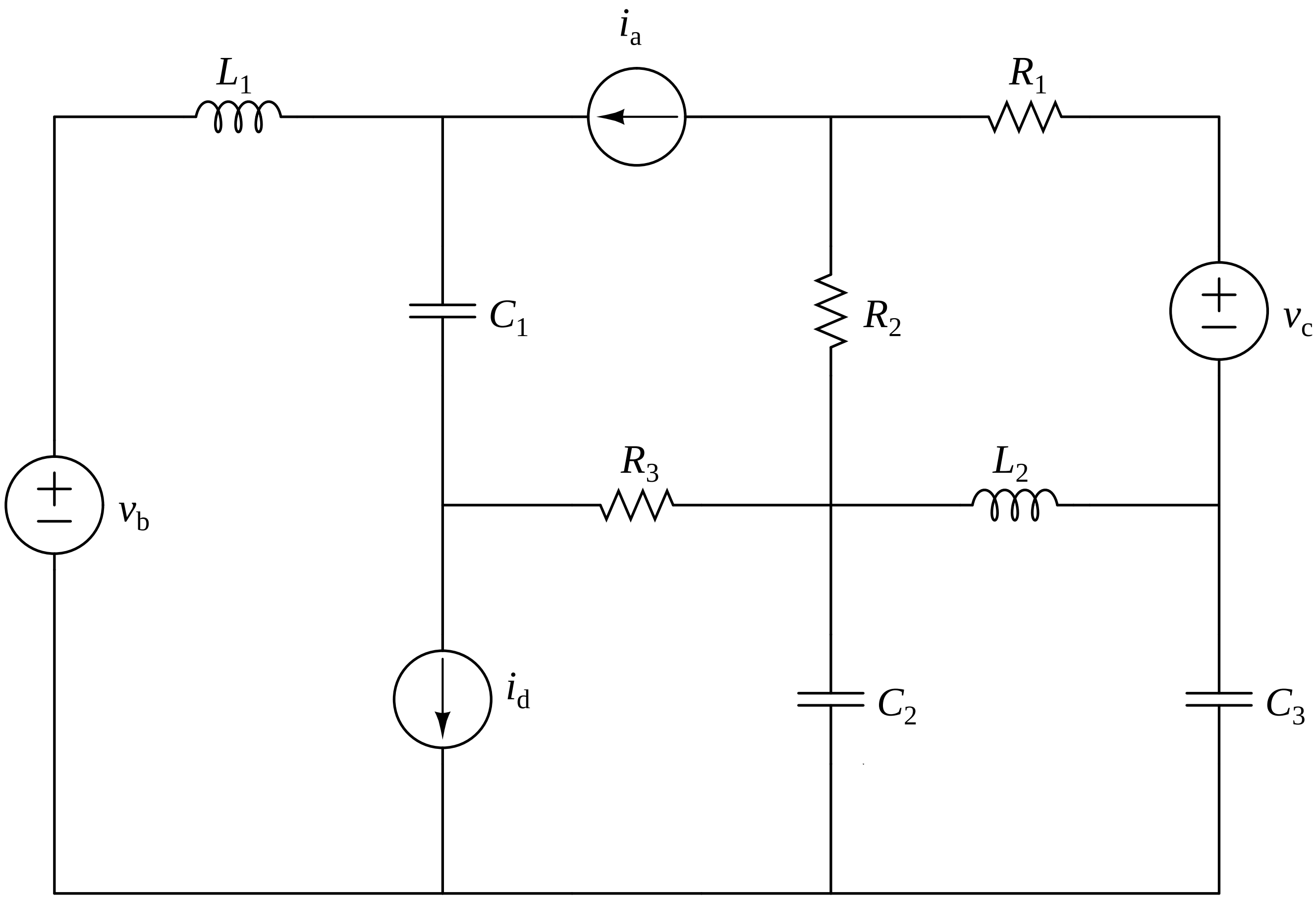
Make sure that your name *and* NET ID are *clearly* written at the top of *every* page, just in case problem parts come loose in the shuffle. Make sure that the work you are submitting for an answer is clearly marked as such. Finally, when turning in the test, individually staple all the work for each problem and place each problem's work in the appropriate folder.

Note that there may be people taking the test after you, so you are not allowed to talk about the test - even to people outside of this class - until I send along the OK. This includes talking about the specific problem types, how long it took you, how hard you thought it was - really anything. Please maintain the integrity of this test.

Name (please print):
Community Standard (print ACPUB ID):

Problem I: [15 pts.] Mesh Current Method

Given the following electrical system:



and assuming that the values for the passive elements (L_1, L_2, R_1, R_2, R_3, C_1, C_2, C_3), and the values for the independent sources (i_a, v_b, v_c, i_d) are known, *clearly* demonstrate the use of the Mesh Current Method in the frequency domain to label unknowns for the circuit and to determine a complete set of equations that could be used to solve for these unknowns. List the set of unknowns you believe your equations will find. You do not need to arrange the equations in matrix format.

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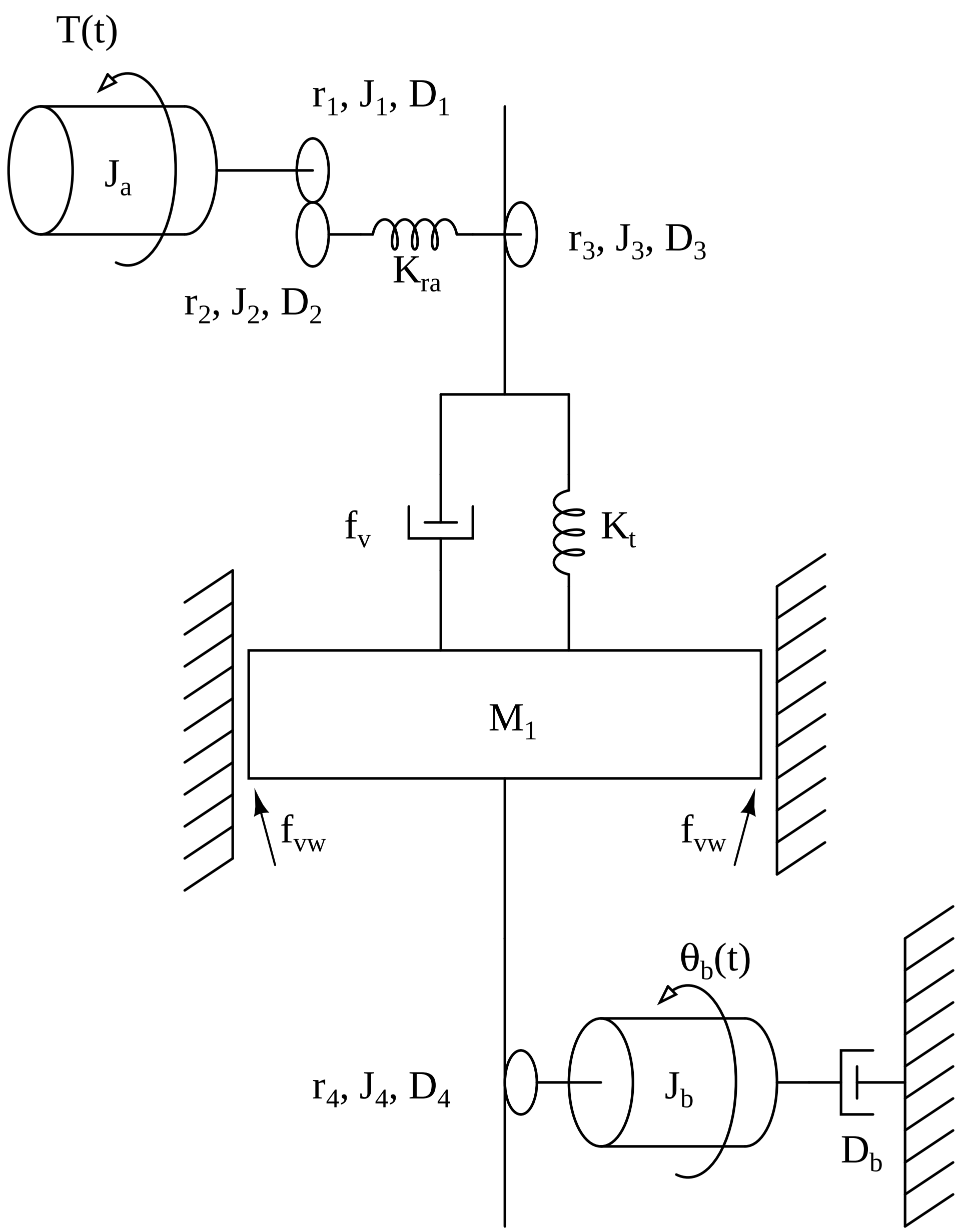
Problem II: [20 pts.] Geared Systems

Given the following system:



where f_{vw} represents viscous friction between the mass and the walls - and is applied to both sides of the mass -

(a) Clearly draw the system from the perspective of the applied torque. Be sure to label any independent angle and also be sure that you indicate how $\theta_{\mathrm{b}}(t)$ relates to your new drawing. As this drawing may be quite wide, you may want to draw along the longer edge of the paper.

(b) Assuming that the values of the passive elements and of the external torque are known, *clearly* determine the equations of (angular) motion for the system in the frequency domain. List the set of unknowns you believe your equations use and be sure they are clearly labeled on the diagram. $\Theta_{\mathrm{b}}(s)$ must be one of your unknowns. You do not need to arrange the equations in matrix format.

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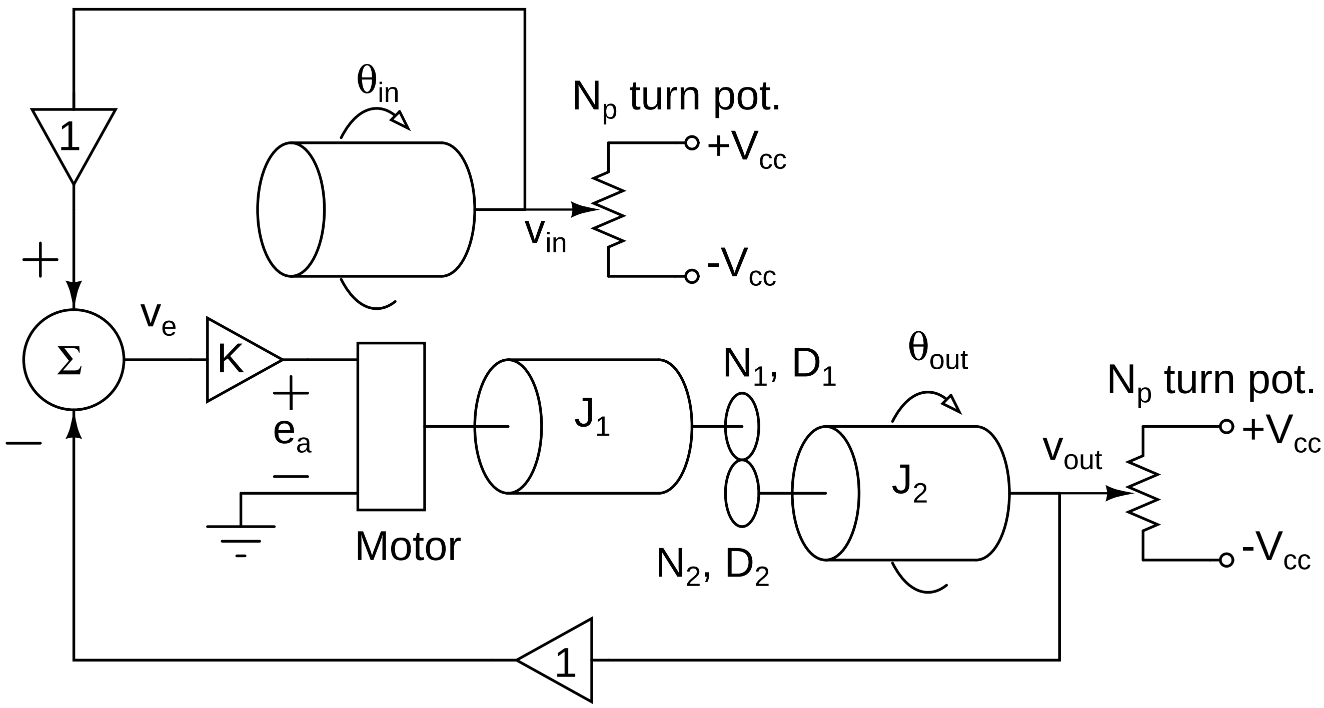
Problem III: [20 pts.] Motor-driven Position Control

The following diagram:



represents a motor-driven position control system. The desired input angle is dialed in using θ_{in} and measured using an N_{p} turn potentiometer. An input angle of 0 rad causes the voltage across the potentiometer to be 0 V. This voltage v_{in} is buffered by an operational amplifier and the connected to the positive input of a difference block. The actual output angle is measured using a similar process with a potentiometer having the same characteristics. The way the system is connected, a positive value of the motor voltage e_{a} will lead to a positive change in the output angle θ_{out}. The motor has characteristic values J_{a}, D_{a}, K_{t}, K_{b}, and R_{a}, which are known.

(1) First, determine the following transfer functions:

(a) $\mathcal{G}_1(s) = \frac{V_{\text{in}}(s)}{\Theta_{\text{in}}(s)}$ - assume that a positive angle $\theta_{\text{in}}(t)$ yields a positive voltage.

(b) $\mathcal{G}_2(s) = \frac{\Theta_{\text{out}}(s)}{V_{\text{e}}(s)}$ - assume that a positive $e_{\text{a}}(t)$ yields a positive $\theta_{\text{out}}(t)$ as drawn. Note that this is the transfer function between the error signal and the output angle. Do not forget the K...

(c) $\mathcal{G}_3(s) = \frac{V_{\text{out}}(s)}{\Theta_{\text{out}}(s)}$ - assume that a positive angle $\theta_{\text{out}}(t)$ yields a positive voltage.

(2) Next, in terms of $\mathcal{G}_1(s)$, $\mathcal{G}_2(s)$, and $\mathcal{G}_3(s)$, write the overall transfer function

$$\mathcal{T}(s) = \frac{\Theta_{\text{out}}(s)}{\Theta_{\text{in}}(s)}$$

You do not need to substitute in your expressions for the $\mathcal{G}_i(s)$ into this - leave it in terms of the $\mathcal{G}_i(s)$.

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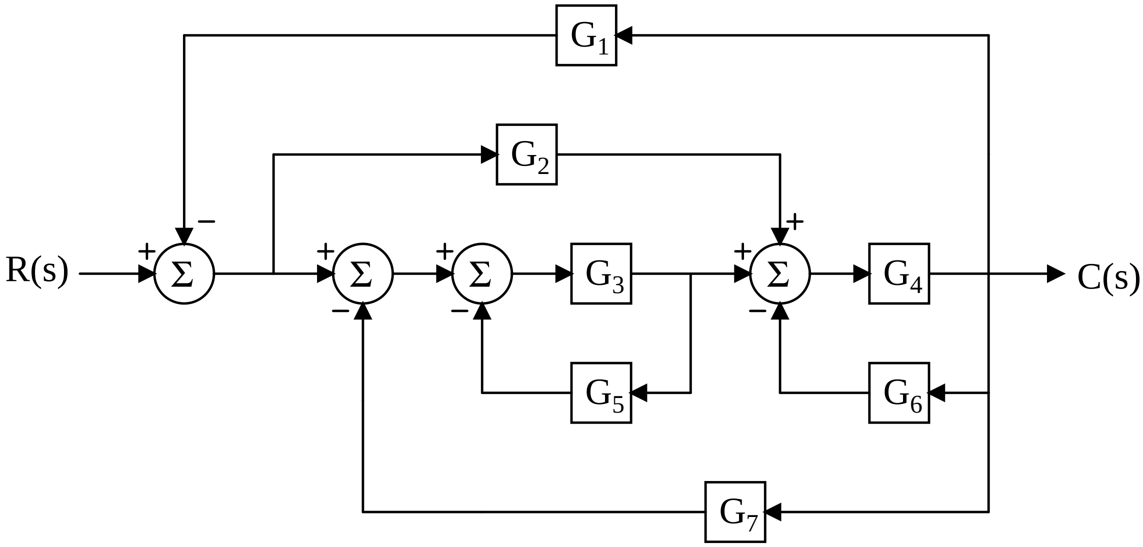
Problem IV: [15 pts.] System Simplification

Given the system below:



(a) Clearly draw a signal flow diagram for the system. Be sure to indicate where each node is on the system.

(b) Use Mason's Rule to determine the overall transfer function $T(s) = C(s)/R(s)$. Note that once you define components in terms of the individual transfer functions, you do *not* need to simplify nor do you need to substitute them into the final result.

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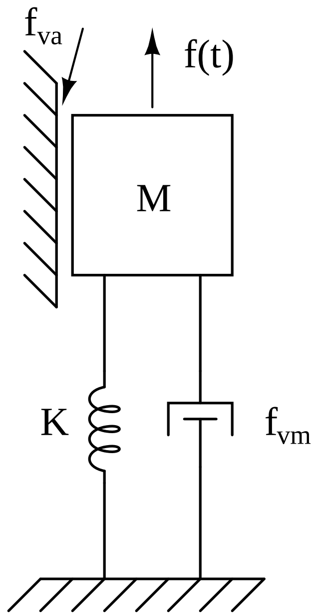
Problem V: [15 pts.] System Design and Characterization

A bobblehead doll can be modeled as a mass attached by a spring to a fixed surface. Though an ideal bobblehead would bounce forever, there is some damping in the system such that a better model would be:



where f_{va} represents the damping caused by air and f_{vm} represents damping caused by the mechanical connections in the systems.

A particular bobblehead doll - made for former Dean Kristina Johnson of the Pratt School of Engineering - is experimentally determined to have a damped frequency of oscillation of 5 Hz and a settling time of about 4 seconds. The top of the doll weighs about 50 grams. Given that information:

(a) Write a symbolic transfer function between the vertical position of the mass and the applied external force.

(b) Determine the numerical values for the spring constant and the *total* effective damping $f_{\text{va}} + f_{\text{vm}}$.

(c) Assuming a step input, determine the rise time for the system.

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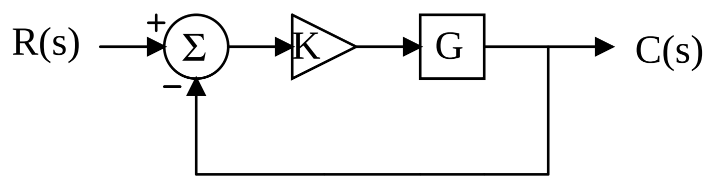
Problem VI: [15 pts.] System Stability Analysis

(1) A plant is found to have a transfer function of:

$$G(s) = \frac{1}{s^3 + 7s^2 + 7s - 15}$$

Clearly, and by hand, generate a Routh array for the plant and determine the general location of the poles of the uncontrolled system. Is this plant stable?

(2) The plant is now controlled using a gain K as a controller in conjunction with negative unity feedback. That is to say:



Given this:

(a) Clearly, and by hand, generate a Routh array for the controlled system.
(b) Determine the range of stability for the controller.
(c) If this system can be marginally stable, determine the gain value(s) for marginal stability and the frequency of oscillation at marginal stability. If this system cannot be marginally stable, explain why you believe that.