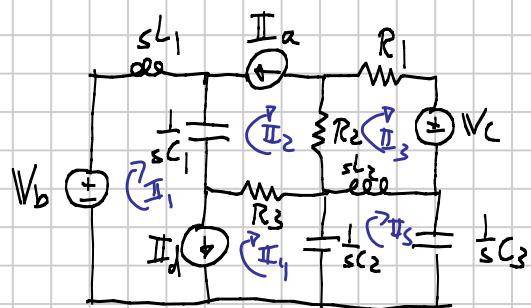


Controls Test | Spring 2011

Note Title

1)



5 unknowns: $\mathbb{I}_1, \mathbb{I}_2, \mathbb{I}_3, \mathbb{I}_4, \mathbb{I}_s$

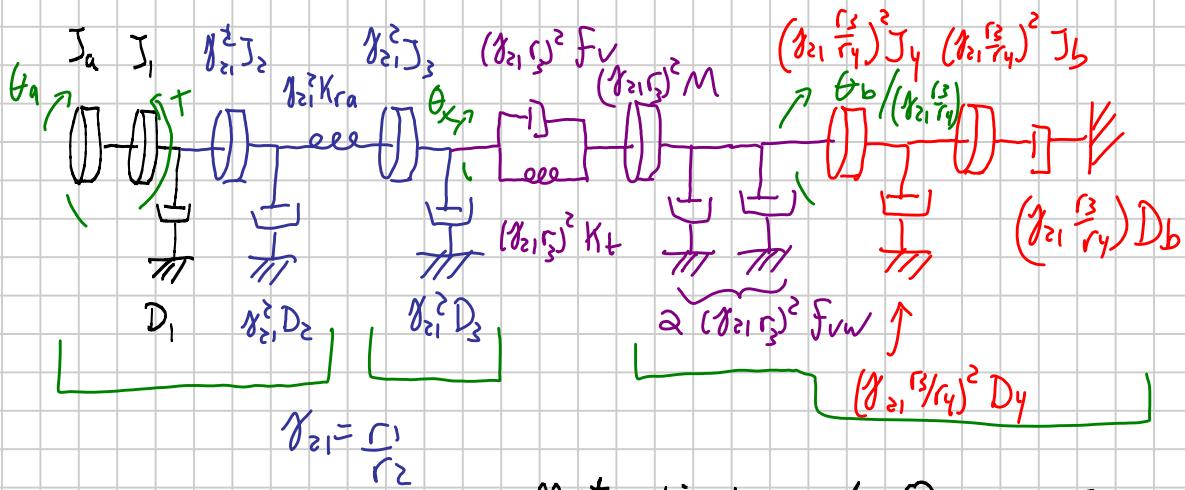
$$KVL_{1, sL_1} -V_b + sL_1 \mathbb{I}_1 + \frac{1}{sC_1} (\mathbb{I}_1 - \mathbb{I}_2) + R_3 (\mathbb{I}_4 - \mathbb{I}_2) + \frac{1}{sC_2} (\mathbb{I}_1 - \mathbb{I}_s) = 0$$

$$KVL_{1, R_3} R_2 (\mathbb{I}_3 - \mathbb{I}_2) + R_1 \mathbb{I}_3 + V_c + sL_2 (\mathbb{I}_3 - \mathbb{I}_s) = 0$$

$$KVL_{1, sC_2} \frac{1}{sC_2} (\mathbb{I}_s - \mathbb{I}_1) + sL_2 (\mathbb{I}_s - \mathbb{I}_3) + \frac{1}{sC_3} \mathbb{I}_s = 0$$

$$\text{AUX } a \quad \mathbb{I}_a = -\mathbb{I}_2$$

$$\text{AUX } d \quad \mathbb{I}_d = \mathbb{I}_1 - \mathbb{I}_4$$



3 DOF

Note direction of Θ_b vs. T
 Θ_a and Θ_x additional unknowns

$$\begin{aligned}
 1) & \quad ((J_a + J_1 + \gamma_{21}^2 J_2) s^2 + (D_1 + \gamma_{21}^2 D_2) s + (\gamma_{21}^2 K_{ra})) \textcircled{H}_a - ((\gamma_{21}^2 K_{ra})) \textcircled{H}_X = -T \\
 2) & \quad -((\gamma_{21}^2 K_{ra})) \textcircled{H}_a + ((\gamma_{21}^2 J_3) s^2 + (\gamma_{21}^2 D_3 + (\gamma_{21} r_3)^2 F_V) s + (\gamma_{21}^2 K_{ra} + (\gamma_{21} r_3)^2 K_T)) \textcircled{H}_X \\
 & \quad - ((\gamma_{21} r_3)^2 F_V) s + (\gamma_{21} r_3)^2 K_T \textcircled{H}_b \frac{\textcircled{H}_b}{(\gamma_{21} r_3/r_4)} = 0 \\
 3) & \quad - ((\gamma_{21} r_3)^2 F_V) s + (\gamma_{21} r_3)^2 K_T \textcircled{H}_X + \\
 & \quad [((\gamma_{21} r_3)^2 M + (\gamma_{21} r_3/r_4)^2 (J_1 + J_B)) s^2 + ((\gamma_{21} r_3)^2 (F_V + 2F_{VW}) + (\gamma_{21} r_1)^2 (D_4 + D_B)) s + \\
 & \quad (\gamma_{21} r_3)^2 K_T) \textcircled{H}_b \frac{\textcircled{H}_b}{(\gamma_{21} r_3/r_4)} = 0
 \end{aligned}$$

3)

a) $\frac{V_{in}}{\Theta_{out}} = \frac{2 V_{cc}}{N_p \text{ turns} \cdot \frac{2\pi \text{ rad}}{\text{turn}}} = \frac{V_{cc}}{\pi N_p} \left[\frac{V}{\text{rad}} \right]$

total voltage change
total angle change

b) $J_{\text{total}} = J_a + J_1 + \gamma_{z1}^2 J_2 \quad \gamma_{z1} = N_1/N_2$

$$D_{\text{total}} = D_a + D_1 + \gamma_{z1}^2 D_2$$

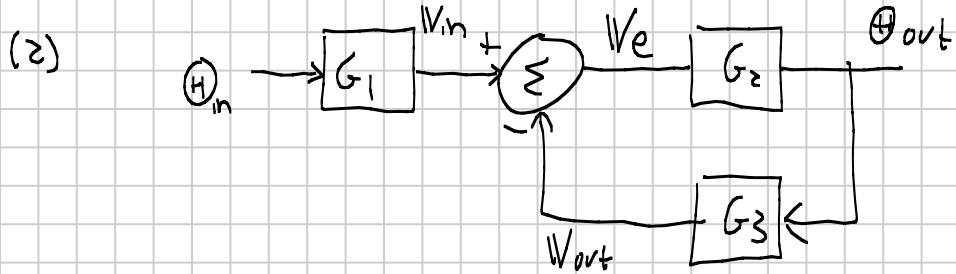
$$\frac{\Theta_{out}}{E_a} = \frac{N_1}{N_2} \Theta_m \quad E_a = K V_e$$

$$\frac{\Theta_m}{E_a} = \frac{\frac{1}{J_{\text{total}}} \frac{K+}{R_a}}{S \left(S + \frac{1}{J_{\text{total}}} \left(D_{\text{total}} + \frac{K+K_b}{R_a} \right) \right)}$$

so

$$G_2 = \frac{K \frac{N_1}{N_2} \frac{1}{J_{\text{total}}} \frac{K+}{R_a}}{S \left(S + \frac{1}{J_{\text{total}}} \left(D_{\text{total}} + \frac{K+K_b}{R_a} \right) \right)}$$

c) $G_3 = \frac{V_{cc}}{\pi N_p}$ as above



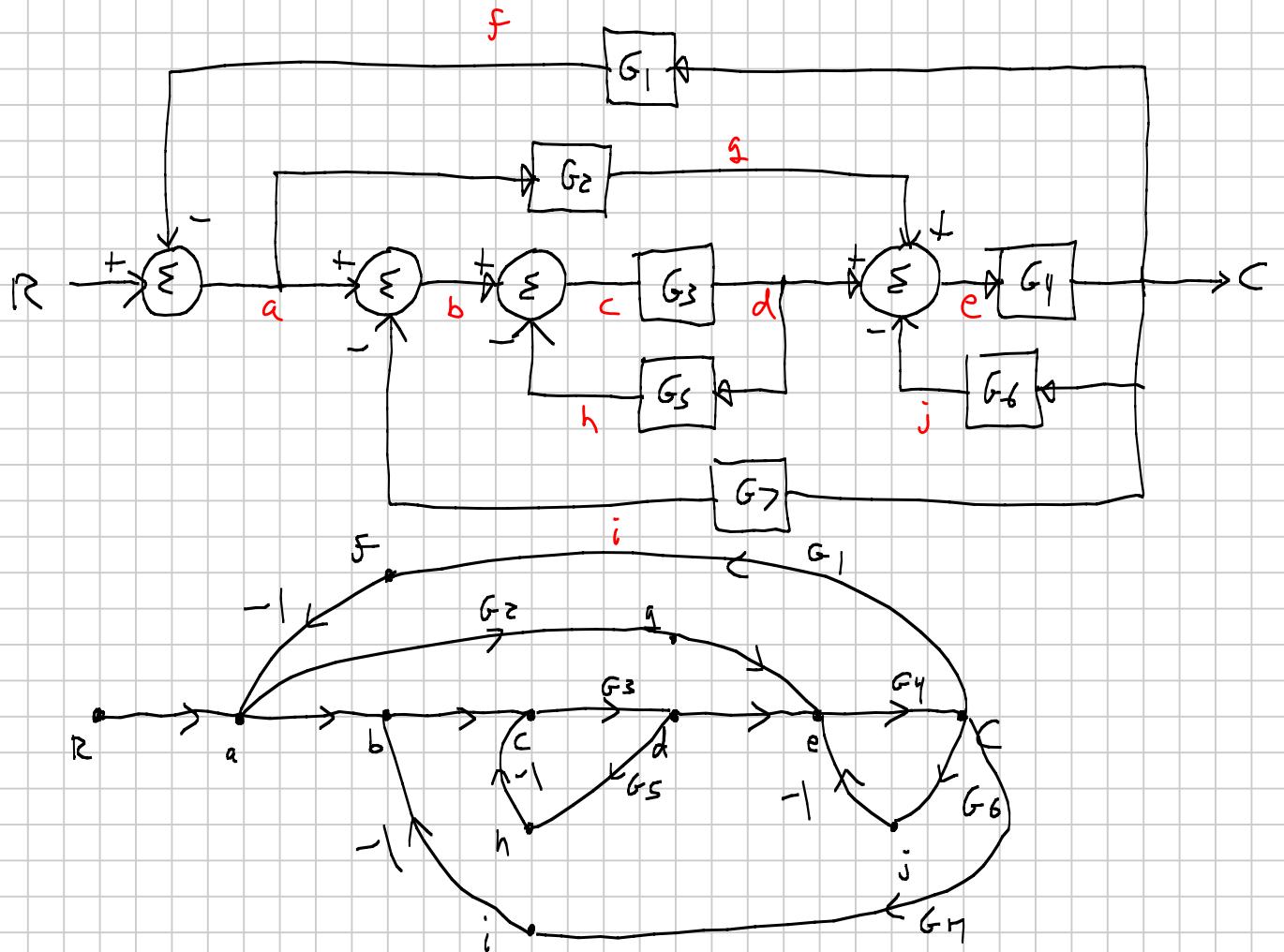
so

$$T = \frac{G_1 G_2}{1 + G_2 G_3}$$

Some also found $\frac{V_{out}}{V_{in}} = \frac{G_2 G_3}{1 + G_2 G_3}$

$$\frac{\Theta_{out}}{\Theta_{in}} = \frac{V_{in}}{\Theta_{in}} \frac{V_{out}}{V_{in}} \frac{\Theta_{out}}{V_{out}} = G_1 \left(\frac{G_2 G_3}{1 + G_2 G_3} \right) \frac{1}{G_3} = \frac{G_1 G_2}{1 + G_2 G_3}$$

* Sample test posted online has some clarifications + a correction of a typographical error.



$$T_1 \quad RabcdeC \quad G_3 G_4 \quad L_1 \quad abcde(Cfa) \quad -G_1 G_3 G_4$$

$$T_2 \quad RaageC \quad G_2 G_4 \quad L_2 \quad a g e(Cfa) \quad -G_1 G_2 G_1$$

$$L_3 \quad b c d e C i b \quad -G_3 G_4 G_7$$

$$L_4 \quad c d h c \quad -G_3 G_5$$

$$L_5 \quad e C j e \quad -G_4 G_6$$

$$\Delta = 1 - L_1 - L_2 - L_3 - L_4 - L_5 + L_2 L_4 + L_4 L_5$$

$$\Delta_1 = 1$$

$$\Delta_2 = 1 - L_4$$

$$T = \frac{T_1 \Delta_1 + T_2 \Delta_2}{\Delta}$$

$$5) (M s^2 + (F_{ra} + F_{rm}) s + K) X = F$$

$$\frac{X}{F} = \frac{1}{Ms^2 + (F_{ra} + F_{rm})s + K} = \frac{1/M}{s^2 + \frac{(F_{ra} + F_{rm})}{M}s + \frac{K}{M}}$$

Must convert Hz \rightarrow rad/s and g \rightarrow kg to have consistent units

$$\omega_d = 2\pi f = 10\pi \text{ rad/s} \quad T_s = 4 \text{ sec}$$

$$T_s = \frac{4}{\zeta \omega_n} = 4 \quad \omega_n = 1 \text{ rad/s}$$

$$* 2\omega_n = \frac{F_{ra} + F_{rm}}{0.05 \text{ kg}} \quad F_{ra} + F_{rm} = (2)(1)(0.05) = 0.1 \text{ N} \frac{\text{s}}{\text{m}}$$

or kg/s

$$\omega_d = \omega_n \sqrt{1 - \zeta^2}$$

$$\omega_d^2 = \omega_n^2 - \omega_n^2 \zeta^2$$

mainly same as ω_d
since ζ is small

$$(10\pi)^2 = \omega_n^2 - 1 \quad \omega_n = \sqrt{1 + (10\pi)^2} = 31.43 \text{ rad/s}$$

$$\frac{K}{M} = \omega_n^2 \quad K = M \omega_n^2 = (0.05)(31.43 \text{ rad/s})^2 = 49.398 \frac{\text{N}}{\text{m}}$$

$$\zeta = \frac{1}{\omega_n} = 0.032$$

or kg/s^2

$$T_r = \frac{1}{\omega_n} \left[1.76\zeta^3 - 0.417\zeta^2 + 1.034\zeta + 1 \right]$$

$$= 0.0329 \text{ sec}$$

$$G = \frac{1}{s^3 + 7s^2 + 7s - 15} \quad \text{note } \frac{1}{(s+3)(s+5)(s-1)}$$

$$\begin{array}{c} L \\ \left(\begin{array}{ccc|cc} s^3 & & & 1 & 7 \\ s^2 & & & -7 & -15 \end{array} \right) \\ L \\ \left(\begin{array}{ccc|cc} s^3 & & & 1 & 7 \\ s^1 & -15-49 & \xrightarrow{\rightarrow} & 64 & 7 \end{array} \right) \\ R \\ \left(\begin{array}{ccc|cc} s^0 & -15 & & & \end{array} \right) \end{array}$$

2 L, 1 R
met stable

$$(2) T = \frac{KG}{1+KG} = \frac{K}{s^3 + 7s^2 + 7s - 15 + K}$$

$$\begin{array}{ccc|c} s^3 & & & 7 \\ s^2 & & & -7 \\ & & & K-15 \end{array}$$

$$K < 64 \quad s^1 \quad \xrightarrow{\frac{K-15-49}{7}} \quad \frac{64-K}{7}$$

$$K > 15 \quad s^0 \quad K-15$$

$$\underline{s^0} \quad 15 < K < 64$$

Marginal \rightarrow if $K=15$, $\frac{49}{7}s^1=0$, $s=0$ rad/s

\downarrow

if $K=64$, $7s^2 + 49s^0 = 0$

$$s = \sqrt{-7} \quad \omega = \sqrt{7} \quad \text{rad/s}$$

Note: the $K=15$ case is not trivial here; this system will have a bounded response to bounded inputs having no DC component.