Duke University
Edmund T. Pratt, Jr. School of Engineering

Controls Fall 2024
Test III Closed Book
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Name (please print):

NetID (please print):

Submitting your work for a grade implies agreement with the following: In keeping with the Community Standard, I have neither provided nor received any assistance on this test. I understand if it is later determined that I gave or received assistance, I will be brought before the Undergraduate Conduct Board and, if found responsible for academic dishonesty or academic contempt, fail the class. I also understand that I am not allowed to communicate with anyone except the instructor about any aspect of this test until the instructor announces it is allowed. I understand if it is later determined that I did communicate with another person about the test before the instructor said it was allowed, I will be brought before the Undergraduate Conduct Board and, if found responsible for academic dishonesty or academic contempt, fail the class.

Instructions

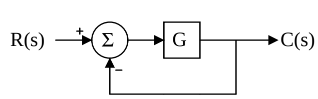
First - please turn **off** any cell phones or other annoyance-producing devices. Vibrate mode is not enough - your device needs to be in a mode where it will make no sounds during the course of the test, including the vibrate buzz or those acknowledging receipt of a text or voicemail.

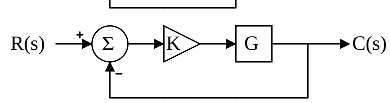
Please be sure that your name and NetID are clearly written at the top of every page. Each problem is on the back of a test page; there is space for work for that particular problem on the front of the next page. **Please be sure to put the work and answers for a given problem on the appropriate page.** Draw a box / circle / something around your final answers for each part of each problem. If you absolutely need more space for a particular problem than the extra nearly-blank page allows, put that work on its own piece of paper, clearly write your name, NetID, and the problem number (in either Arabic or Roman numerals) at the **top center** of that page, and submit any such extra pages in problem-order **after** all preprinted pages of the test. Also, on the pages given to work on a problem, write a note that says "see extra page."

You will *not* be stapling your test but instead will be turning in your test in its original folder to the box at the front of the room. Carefully stack the test pages in order (with any additional pages properly labeled and **after all the original test pages**), put them in the folder you received with the test, and bring the folder to the front of the room.

Note that there may be people taking the test after you, so you are not allowed to talk about the test - even to people outside of this class - until I send along the OK. This includes talking about the specific problem types, how long it took you, how hard you thought it was - really anything. Please maintain the integrity of this test.

In all problems, $u(t)$ is the unit step and $\delta(t)$ is the unit impulse. Also, transfer functions labeled T are generally overall transfer functions while G and H are generally subordinate forward and feedback transfer functions, respectively. Finally, the following represent the block diagrams for unity feedback systems and unity feedback systems with cascaded proportional (gain) control, respectively:





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Problem I: [20 *pts.*] System Categorization and Gain Control I

Given a unity feedback system with a gain controller K and a plant with a transfer function:

$$Gp(s) = \frac{s+5}{(s-1)(s+2)} = \frac{s+5}{s^2+s-2}$$

and an overall system transfer function of:

$$T(s) = \frac{(s+5)K}{s^2+(K+1)s+5K-2}$$

1. Determine the range of stability for the gain controller.
2. Accurately sketch the root locus assuming gain control with $K > 0$. Be sure to denote any interesting locations such as open loop poles, zeros, marginal stability frequencies, and centers of asymptotes. Clearly label these points along with their values. Note that a break point analysis can be used to determine that $\frac{dK}{d\sigma} = 0$ at (K=0.5147, σ=-0.7574) and at (K=17.49, σ=-9.243) - show where and how you used that information on the root locus plot.
3. What are the K values for which the system's dominant poles are overdamped?
4. What are the K values for which the system's dominant poles are critically damped?
5. What are the K values for which the system's dominant poles are underdamped?
6. With respect to steady-state error, determine the system type, the appropriate static error constant, and the steady-state error assuming each of the following inputs and further assuming that the gain is set such that the response has its fastest settling time.
   (a) $r(t) = u(t)$
   (b) $r(t) = tu(t)$
   (c) $r(t) = \frac{1}{2}t^2u(t)$

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Extra work for Problem I

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Problem II: [30 *pts.*] System Categorization and Gain Control II

Given a unity feedback system with a gain controller K and a plant with a transfer function:

$$Gp(s) = \frac{1}{(s)(s+8)(s+12)} = \frac{1}{s^3 + 20s^2 + 96s}$$

and an overall system transfer function of:

$$T(s) = \frac{K}{s^3 + 20s^2 + 96s + K}$$

1. Determine the range of stability for the gain controller.
2. Accurately sketch the root locus assuming gain control with $K > 0$. Be sure to denote any interesting locations such as open loop poles, zeros, marginal stability frequencies, and centers of asymptotes. Clearly label these points along with their values. Note that a break point analysis can be used to determine that $\frac{dK}{d\sigma} = 0$ at (K=-40.39, σ=-10.19) and at (K=135.2, σ=-3.138) - show where and how you used that information on the root locus plot.
3. What are the K values for which the system's dominant poles are overdamped?
4. What are the K values for which the system's dominant poles are critically damped?
5. What are the K values for which the system's dominant poles are underdamped?
6. What is the fastest settling time this controlled system can achieve? At that control point, give the control value, the location of the dominant poles, and the approximate settling time.
7. With respect to steady-state error, determine the system type for $K > 0$, the appropriate static error constant, and the steady-state error assuming each of the following inputs and further assuming that the gain is set such that the response is critically damped.
   (a) $r(t) = u(t)$
   (b) $r(t) = tu(t)$
   (c) $r(t) = \frac{1}{2}t^2u(t)$

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Extra work for Problem II

Chapter 2

Variations on Final Value Theorem (Table 2.2, p. 37)

If $T(s) = \frac{C(s)}{R(s)}$, $c(\infty) = \lim_{s\to 0} sC(s) = \lim_{s\to 0} sR(s)T(s)$
If $T(s) = \frac{C(s)}{R(s)}$ and $r(t) = u(t)$ such that $R(s) = \frac{1}{s}$, $c(\infty) = \lim_{s\to 0} sR(s)T(s) = \lim_{s\to 0} T(s)$

Chapter 4

First Order Characteristics (4.9 & 10, p. 163)

$$T_{\mathrm{r}} = \frac{2.2}{a} \qquad T_{\mathrm{s}} = \frac{4}{a}$$

Underdamped Second Order Characteristics (4.34, 38, 39, 42, and 44 , pp. 175-176)

$$T_{\mathrm{p}} = \frac{\pi}{\omega_n\sqrt{1-\zeta^2}} = \frac{\pi}{\omega_d} \qquad \%OS = e^{-(\zeta\pi/\sqrt{1-\zeta^2})} \times 100 \qquad \zeta = \frac{-\ln(\%OS/100)}{\sqrt{\pi^2 + \ln^2(\%OS/100)}} \qquad T_{\mathrm{s}} \approx \frac{4}{\zeta\omega_{\mathrm{n}}}$$

Underdamped Second Order Rise Time (footnote 5, p. 177)

$$\omega_{\mathrm{n}} T_{\mathrm{r}} = 1.76\zeta^3 - 0.417\zeta^2 + 1.039\zeta + 1,\ 0 < \zeta < 0.9$$

Chapter 7

Static Error Constants and Errors

$$K_{\mathrm{p}} = \lim_{s\to 0} G_{\mathrm{eq}}(s) \quad e_{\mathrm{step}}(\infty) = \frac{1}{1+K_{\mathrm{p}}} \quad K_{\mathrm{v}} = \lim_{s\to 0} sG_{\mathrm{eq}}(s) \quad e_{\mathrm{ramp}}(\infty) = \frac{1}{K_{\mathrm{v}}} \quad K_{\mathrm{a}} = \lim_{s\to 0} s^2 G_{\mathrm{eq}}(s) \quad e_{\mathrm{parabola}}(\infty) = \frac{1}{K_{\mathrm{a}}}$$

Steady State Error Via Input R and Disturbance D (7.61, 7.62, pp. 351-352)

For a unity feedback system with controller $G_1(s)$ and plant $G_2(s)$:

$$e(\infty) = \lim_{s\to 0} sE(s) = \lim_{s\to 0} \frac{s}{1+G_1(s)G_2(s)} R(s) - \lim_{s\to 0} \frac{sG_2(s)}{1+G_1(s)G_2(s)} D(s) = e_{\mathrm{R}}(\infty) + e_{\mathrm{D}}(\infty)$$

$$e_{\mathrm{D,step}}(\infty) = -\frac{1}{\lim_{s\to 0} \frac{1}{G_2(s)} + \lim_{s\to 0} G_1(s)}$$

General

Unity Feedback Equations; Polynomial Ratio Simplifications

$$G_{\mathrm{eq}} = \frac{T}{1-T} \qquad T = \frac{N_{\mathrm{T}}}{D_{\mathrm{T}}} \qquad G_{\mathrm{eq}} = \frac{N_{\mathrm{T}}}{D_{\mathrm{T}} - N_{\mathrm{T}}}$$

$$T = \frac{G_{\mathrm{eq}}}{1+G_{\mathrm{eq}}} \qquad G_{\mathrm{eq}} = \frac{N_{\mathrm{G}}}{D_{\mathrm{G}}} \qquad T = \frac{N_{\mathrm{G}}}{D_{\mathrm{G}} + N_{\mathrm{G}}}$$