(1) $T = \frac{s+100}{(s+1)(s+20)}$

$c_{ss} = \lim_{s\to 0} sRT = \lim_{s\to 0} T = \frac{100}{20} = 5$

$R = \frac{1}{s}$

dom. pole at -1, $T_s = 4/1 = 4$ sec

(2) $G = \frac{4}{s+2}$ $T = \frac{G}{1+G} = \frac{4}{s+6}$

$c_{ss} = \lim_{s\to 0} sRT = \lim_{s\to 0} T = \frac{2}{3}$

$R = \frac{1}{s}$

dom pole at -6, $T_s = 4/6 = 2/3$

(3) $T = \frac{50}{s^2+8s+40}$ $c_{ss} = \lim_{s\to 0} sRT = \lim_{s\to 0} T = \frac{50}{40} = 1.25$

$R = 1/s$

$\omega_n = \sqrt{40} = 2\sqrt{10} = 6.325$

$2\zeta\omega_n = 8$ $\zeta = \frac{4}{\omega_n} = \frac{2}{\sqrt{10}} = \frac{2\sqrt{10}}{10} = \frac{\sqrt{10}}{5} = 0.6325$

$T_s = \frac{4}{\zeta\omega_n} = \frac{4}{4} = 1$ $\%OS = e^{(-\zeta\pi/\sqrt{1-\zeta^2})} \times 100 = 7.69\%$ $\omega_d = \sqrt{40}\sqrt{1-\frac{4}{10}}$ $\sqrt{40}\sqrt{\frac{6}{10}}$ $\sqrt{24}$ $2\sqrt{6}$

$\omega_d = \omega_n\sqrt{1-\zeta^2} = 4.899$

1.25	1	7.69%
6.325	0.6325	4.899

(4) $T = \frac{G}{1+G} = \frac{20}{s^2+5s+26}$ $c_{ss} = \lim_{s\to 0} sRT = \lim_{s\to 0} T = \frac{20}{26} = 0.769$

$R = \frac{1}{s}$

$\omega_n = \sqrt{26} = 5.099$

$2\zeta\omega_n = 5$ $\zeta = \frac{5}{2\cdot\sqrt{26}} = 0.490$

$T_s = \frac{4}{\zeta\omega_n} = \frac{4}{2.5} = 1.6$ $\%OS = e^{(-\zeta\pi/\sqrt{1-\zeta^2})} \times 100 = 17.08\%$

$\omega_d = \omega_n\sqrt{1-\zeta^2} = 4.444$

0.769	1.6	17.08%
5.099	0.490	4.444

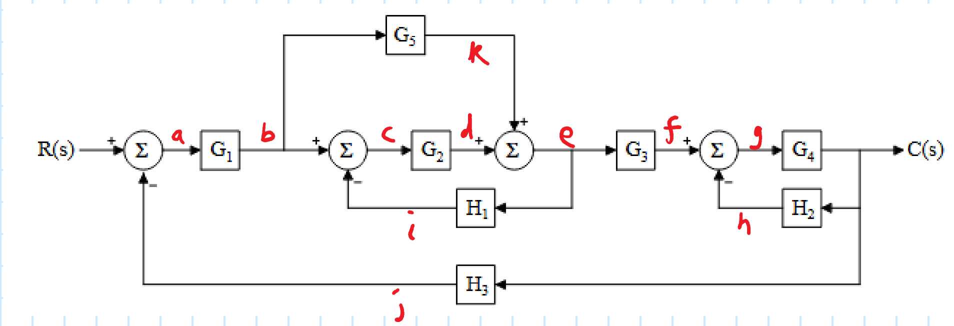
(5) $c_{ss} = 0.8$ $\%OS = 10$ $T_s = 5 = \frac{4}{\zeta\omega_n}$ $\zeta\omega_n = 0.8$

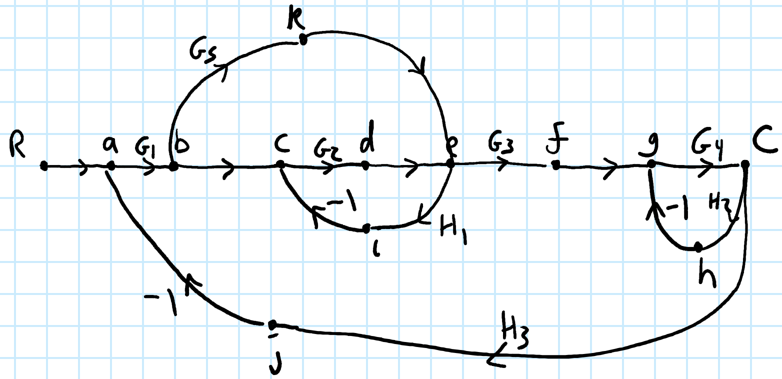
$\zeta = \frac{-\ln(.1)}{\sqrt{\pi^2+\ln^2(.1)}} = 0.5912$ $\omega_n = \frac{0.8}{\zeta} = 1.353$

$\omega_d = \omega_n\sqrt{1-\zeta^2} = 1.091$ $\sigma = -\zeta\omega_n \pm j\omega_d$

$= -0.8 \pm j1.091$

2





$T_1: RabcdefgC = G_1G_2G_3G_4$

$T_2: RabkefgC = G_1G_5G_3G_4$

$L_1 = abcdefgCja = -G_1G_2G_3G_4H_3$

$L_2 = abkefgCja = -G_1G_5G_3G_4H_3$

$L_3: ceic = -G_2H_1$

$L_4: gChg = -G_4H_2$

$\Delta = 1 - L_1 - L_2 - L_3 - L_4 + L_3L_4$

$\Delta_1 = 1$

$\Delta_2 = 1$

$$T = \frac{T_1\Delta_1 + T_2\Delta_2}{\Delta}$$

3

(1)

L	s^5	1	5	-36
NOT R	s^4	4	20	-144
NOT R	s^3	$\cancel{0}\ 16$	$\cancel{0}\ 40$	
NOT R	s^2	10	-144	
R	s^1	270.4		
	s^0	-144		

$$\frac{\begin{vmatrix} 1 & 5 \\ 4 & 20 \end{vmatrix}}{-4} = \frac{20-20}{-4} = 0 \qquad \frac{\begin{vmatrix} 1 & -36 \\ 4 & -144 \end{vmatrix}}{-4} = \frac{-144+144}{-4} = 0$$

$$\frac{d}{dt}\left(4s^4+20s^2-144\right) = 16s^3+40s$$

$$\frac{\begin{vmatrix} 4 & 20 \\ 16 & 40 \end{vmatrix}}{-16} = \frac{160-320}{-16} = 10$$

$$\frac{\begin{vmatrix} 16 & 40 \\ 10 & -144 \end{vmatrix}}{-10} = \frac{-2304-400}{-10} = 270.4$$

1R so 1L + 2jω + 1L = 2L, 1R, 2jω

2) $T = \dfrac{N}{D+N} = \dfrac{K(s+2)}{s^3+s^2+(K-30)s+(2K-72)}$

(a)

s^3	1	$K-30$
s^2	1	$2K-72$
s^1	$42-K$	
s^0	$2K-72$	

$$\frac{\begin{vmatrix} 1 & K-30 \\ 1 & 2K-72 \end{vmatrix}}{-1} = \frac{2K-72-K+30}{-1} = \frac{K-42}{-1}$$

$42-K>0 \quad K<42$

$2K-72>0 \quad K>36$

$$\boxed{36<K<42}$$

(b) $K=36$ YIELDS $s=0$ $\qquad \omega = 0$ rad/s

$K=42$ YIELDS $s^2+(2\cdot 42-72) = s^2+12 = 0 \quad s = \pm j\sqrt{12} = \pm j3.464$

$\omega = 3.464$ rad/s

(c) TYPE 0 (NO s IN DENOM.)

(d) $K_p = \lim\limits_{s\to 0} G = \dfrac{2K}{-72} = -\dfrac{K}{36}$

$e_{step} = \dfrac{1}{1+K_p} = \dfrac{1}{1-\frac{K}{36}} = \dfrac{36}{36-K}$

$KG = \frac{K(s+7)}{(s)(s^2+6s+13)}$ $\quad T = \frac{KG}{1+KG} = \frac{K(s+7)}{s^3+6s^2+(K+13)s+7K}$

(1) MARGINALLY STABLE (poles at $s = \underline{0}, -3 \pm j2$)

(2)

s^3	1	$K+13$
s^2	6	$7K$
s^1	$\frac{78-K}{6}$	
s^0	$7K$	

$\frac{\begin{vmatrix} 1 & K+13 \\ 6 & 7K \end{vmatrix}}{-6} = \frac{7K-6K-78}{-6} = \frac{78-K}{6}$

$K < 78$

$K > 0$

$0 < K < 78$

(3) TYPE 1

(4) K_v $\quad e_{ramp}(\infty)$

(5) $K_v = \lim_{s \to 0} sG(s) = \frac{7K}{13}$

$e_{ramp} = \frac{1}{K_v} = \frac{13}{7K}$

(6) $K \to 78$ $\quad e_{ramp} = \frac{13}{7 \times 78} = 0.02381$

(7) $-0.544 \pm j5.04$ are dominant (slowest)

underdamped (dominant are complex)

$$T = \frac{K(s+4)}{s^3+7s^2+(K-34)s+4K-40}$$

$$G = \frac{T}{1-T} = \frac{K(s+4)}{s^3+7s^2-34s-40}$$

1)

s^3	1	$K-34$
s^2	7	$4K-40$
s^1	$\frac{3K-198}{7}$	
s^0	$4K-40$	

$$\frac{\begin{vmatrix} 1 & K-34 \\ 7 & 4K-40 \end{vmatrix}}{-7} = \frac{4K-40-7K+238}{-7} = \frac{3K-198}{7}$$

$K > \frac{198}{3}$ $\quad K > 66$

$K > 10$

2) TYPE 0

3) K_p, $e_{step}(\infty)$

4) $K_p = \lim_{s \to 0} G(s) = \frac{4K}{-40} = \frac{-K}{10}$

$e_{step} = \frac{1}{1+K_p} = \frac{1}{1-\frac{K}{10}} = \frac{10}{10-K}$

5) $K = 66$

6) $7s^2 + (4 \cdot 66 - 40) = 7s^2 + 224 = 0 \quad s^2 = -32 \quad s = \pm j\sqrt{32} = \pm j4\sqrt{2} = \pm j5.657$

$\omega = 4\sqrt{2} = 5.657$