(1) $a(t) = (\sin(2t) + 3te^{-4t})u(t)$

$\mathcal{A}(s) = \frac{2}{s^2+4} + \frac{3}{(s+4)^2}$

(2) $\mathcal{B}(s) = \frac{2s+7}{s^2+6s+18} = \frac{2s+7}{(s+3)^2+(3)^2} = \frac{A(s+3)+B(3)}{(s+3)^2+(3)^2}$

$b^2-4ac = 36-72 < 0$

MOAT $\quad A=2 \quad 6+3B=7 \quad B=1/3$

$b(t) = e^{-3t}(2\cos(3t) + \frac{1}{3}\sin(3t))u(t)$

(3) $C(s) = \frac{5s+8}{s^2+10s+25} = \frac{5s+8}{(s+5)^2} = \frac{5s+25-25+8}{(s+5)^2} = \frac{5}{(s+5)} - \frac{17}{(s+5)^2}$

$b^2-4ac = 100-100=0$

repeat

$c(t) = (5e^{-5t} - 17te^{-5t})u(t)$

(4) $x(t) = 2\cos(2t)e^{-t}u(t) \qquad y(t) = \sin(2t)e^{-t}u(t) + e^{-2t}u(t)$

$\mathcal{X}(s) = \frac{2(s+1)}{(s+1)^2+(2)^2} \qquad \mathcal{Y}(s) = \frac{1(2)}{(s+1)^2+(2)^2} + \frac{1}{s+2} = \frac{(2)(s+2)+(s^2+2s+5)}{((s+1)^2+(2)^2)(s+2)}$

$\mathcal{H}(s) = \frac{\mathcal{Y}(s)}{\mathcal{X}(s)} = \frac{s^2+4s+9}{((s+1)^2+(2)^2)(s+2)} \cdot \frac{(s+1)^2+(2)^2}{2s+2} = \frac{s^2+4s+9}{2s^2+6s+4}$

$(2s^2+6s+4)\mathcal{Y}(s) = (s^2+4s+9)\mathcal{X}(s)$

$2\frac{d^2y(t)}{dt^2} + 6\frac{dy(t)}{dt} + 4y(t) = \frac{d^2x(t)}{dt^2} + 4\frac{dx(t)}{dt} + 9x(t)$

(5) $\frac{d^2y(t)}{dt^2} + 6\frac{dy(t)}{dt} + 8y(t) = 2 \qquad t>0$

$(s^2\mathcal{Y} - sy(0^-) - \dot{y}(0^-)) + 6(s\mathcal{Y} - y(0^-)) + 8\mathcal{Y} = \frac{2}{s}$

$(s^2+6s+8)\mathcal{Y} = sy(0^-) + \dot{y}(0^-) + 6y(0^-) + \frac{2}{s} = 3 + \frac{2}{s} = \frac{3s+2}{s}$

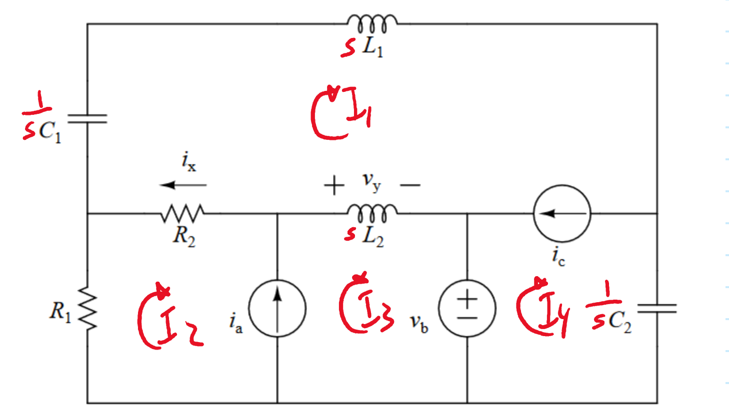
$\mathcal{Y} = \frac{3s+2}{(s^2+6s+8)(s)} = \frac{3s+2}{(s+2)(s+4)(s)} = \frac{A}{s+2} + \frac{B}{s+4} + \frac{C}{s}$

$A = \lim_{s\to-2} \frac{3s+2}{(s+4)(s)} = \frac{-4}{2\cdot-2} = 1 \qquad B = \lim_{s\to-4} \frac{3s+2}{(s+2)(s)} = \frac{-10}{(-2)(-4)} = \frac{-5}{4}$

$C = \lim_{s\to0} \frac{3s+2}{(s+2)(s+4)} = \frac{2}{(2)(4)} = \frac{1}{4}$

$y(t) = \left(e^{-2t} - \frac{5}{4}e^{-4t} + \frac{1}{4}\right)u(t)$

NOTE: $y(0) = 1 - \frac{5}{4} + \frac{1}{4} = 0 \qquad \dot{y}(0) = -2 + 5 + 0 = 3$ ✓



UNK:

$I_1 \; I_2 \; I_3 \; I_4$

$I_x \; V_y$

KVL $_{1 \& 14}$: $\frac{1}{sC_1} I_1 + sL_1 I_1 + \frac{1}{sC_2} I_4 - V_b + sL_2(I_1 - I_3) + R_2(I_1 - I_2) = 0$

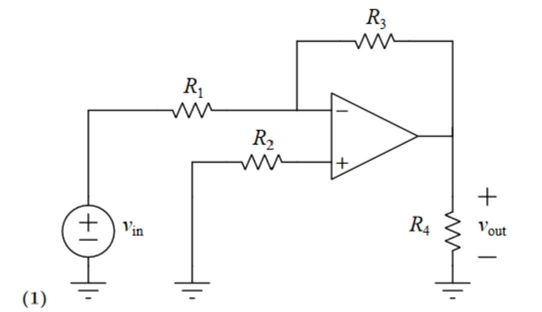
KVL, $s\ell_{23}$: $R_1 I_2 + R_2(I_2 - I_1) + sL_2(I_3 - I_1) + V_b = 0$

SRC I_a: $I_a = I_3 - I_2$

SRC I_c: $I_c = I_1 - I_4$

MEAS I_x: $I_x = I_1 - I_2$

MEAS V_y: $V_y = sL_2(I_3 - I_1)$

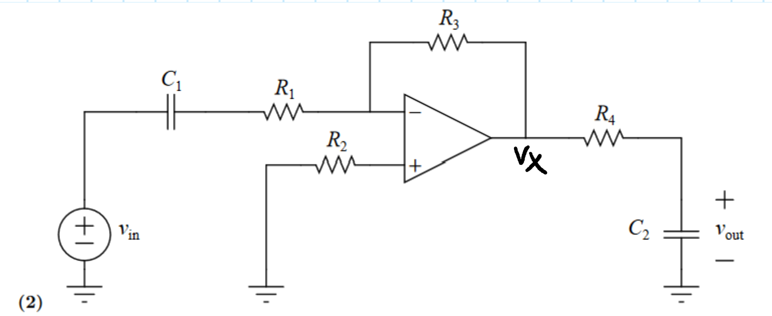


(1)

INVERTING

$Z_f = R_3 \quad Z_n = R_1$

$H = -\frac{Z_f}{Z_n} = -\frac{R_3}{R_1}$



(2)

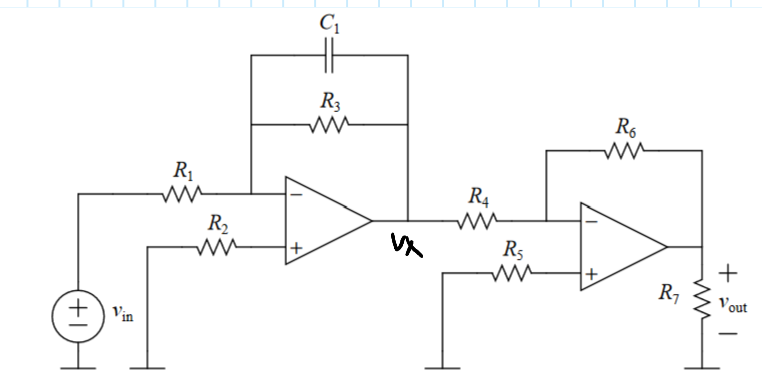
INVERTING

$Z_f = R_3$

$Z_n = \frac{1}{sC_1} + R_1 = \frac{sR_1C_1 + 1}{sC_1}$

$\frac{V_x}{V_{in}} = -\frac{Z_f}{Z_n} = -\frac{sC_1R_3}{sR_1C_1 + 1}$

VOLTAGE DIVIDER: $\frac{V_{out}}{V_x} = \frac{\frac{1}{sC_2}}{R_4 + \frac{1}{sC_2}} = \frac{1}{sR_4C_2 + 1}$; $\frac{V_{out}}{V_{in}} = \frac{V_x}{V_{in}}\frac{V_{out}}{V_x} = -\frac{sC_1R_3}{(sR_1C_1+1)(sR_4C_2+1)}$



(3)

OA 1: INVERTING

$Z_f = \frac{\frac{1}{sC_1} \cdot R_3}{\frac{1}{sC_1} + R_3} = \frac{R_3}{sR_3C_1 + 1} \quad Z_n = R_1$

$\frac{V_x}{V_{in}} = -\frac{Z_f}{Z_n} = -\frac{R_3/R_1}{sR_3C_1 + 1}$

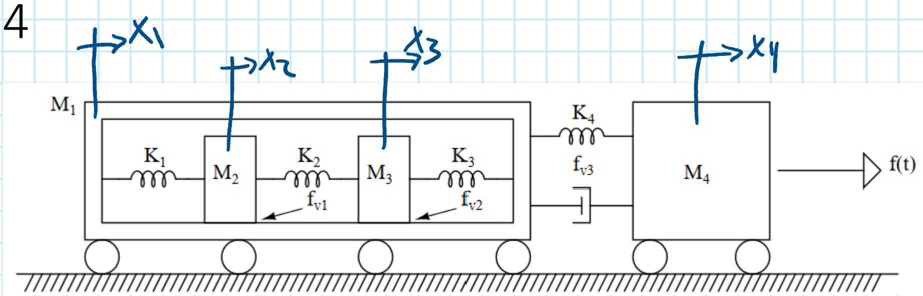
OA 2: INVERTING

$Z_f = R_6 \quad Z_n = R_4$

$\frac{V_{out}}{V_x} = -\frac{Z_f}{Z_n} = -\frac{R_6}{R_4}$

$\frac{V_{out}}{V_{in}} = \frac{V_x}{V_{in}}\frac{V_{out}}{V_x} = \left(-\frac{R_3/R_1}{sR_3C_1+1}\right)\left(-\frac{R_6}{R_4}\right) = \frac{(R_3R_6)/(R_1R_4)}{sR_3C_1 + 1}$

4



UNK: X_1 X_2 X_3 X_4

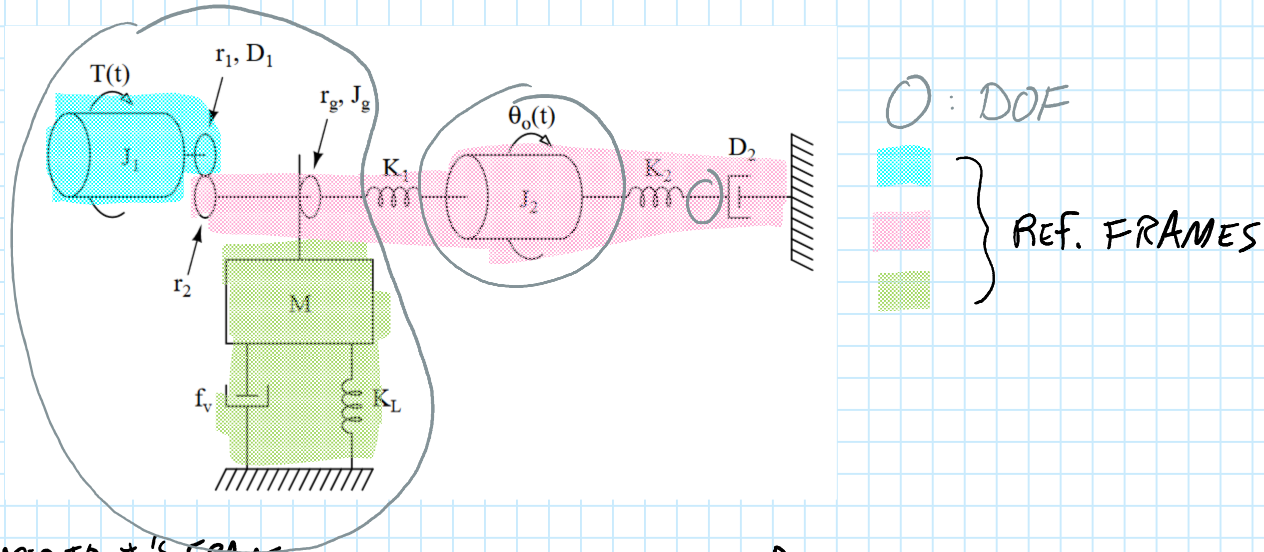
1: $(M_1s^2+(f_{v1}+f_{v2}+f_{v3})s+(K_1+K_3+K_4))X_1-(f_{v1}s+K_1)X_2-(f_{v2}s+K_3)X_3-(f_{v3}s+K_4)X_4=0$

2: $-(f_{v1}s+K_1)X_1+(M_2s^2+f_{v1}s+(K_1+K_2))X_2-K_2X_3=0$

3: $-(f_{v2}s+K_3)X_1-K_2X_2+(M_3s^2+f_{v2}s+(K_2+K_3))X_3=0$

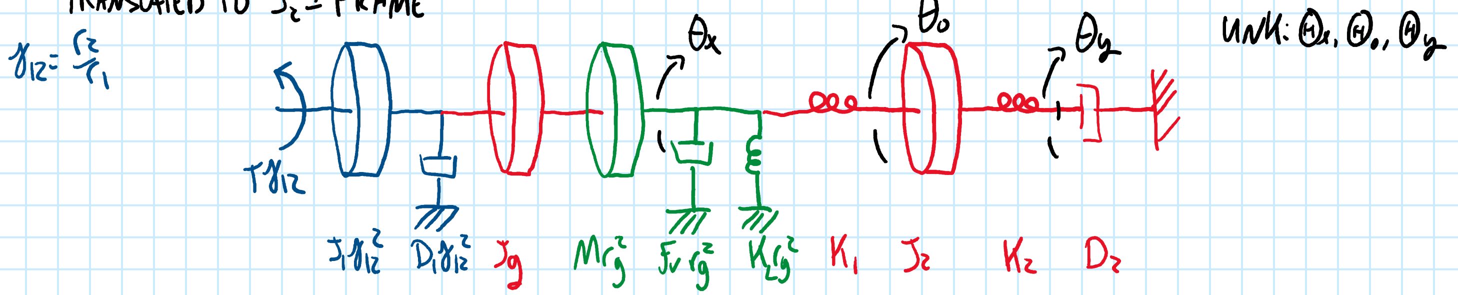
4: $-(f_{v3}s+K_4)X_1+(M_4s^2+f_{v3}s+K_4)X_4=F$

$$\begin{bmatrix} M_1s^2+(f_{v1}+f_{v2}+f_{v3})s+K_1+K_3+K_4 & -(f_{v1}s+K_1) & -(f_{v2}s+K_3) & -(f_{v3}s+K_4) \\ -(f_{v1}s+K_1) & M_2s^2+f_{v1}s+K_1+K_2 & -K_2 & 0 \\ -(f_{v2}s+K_3) & -K_2 & M_3s^2+f_{v2}s+K_2+K_3 & 0 \\ -(f_{v3}s+K_4) & 0 & 0 & M_4s^2+f_{v3}s+K_4 \end{bmatrix}\begin{bmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \end{bmatrix}=\begin{bmatrix} 0 \\ 0 \\ 0 \\ F \end{bmatrix}$$



TRANSLATED TO J_2'S FRAME

$\gamma_{12} = \frac{r_2}{r_1}$



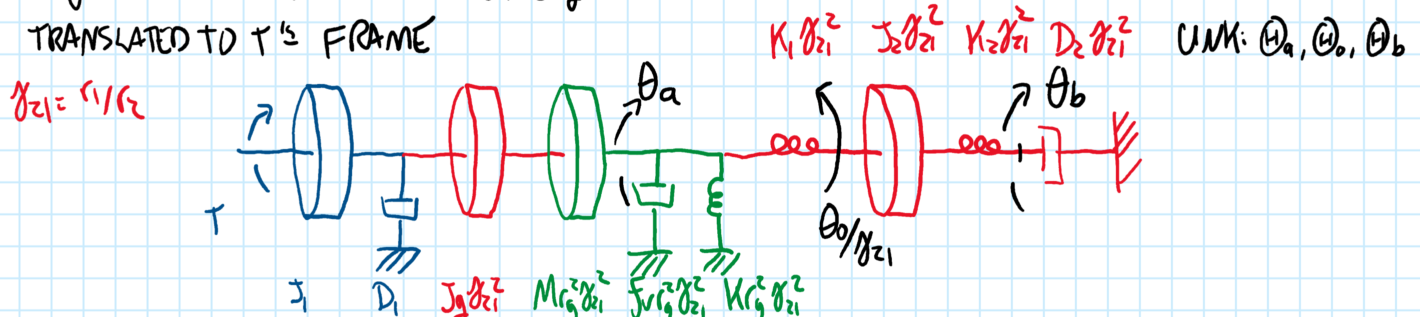
$$x: \quad \left(\left(J_1\gamma_{12}^2 + J_g + M r_g^2\right)s^2 + \left(D_1\gamma_{12}^2 + f_v r_g^2\right)s + \left(K_L r_g^2 + K_1\right)\right)\Theta_x - K_1\Theta_o = -T\gamma_{12}$$

$$o: \quad -K_1\Theta_x + \left(J_2 s^2 + K_1 + K_2\right)\Theta_o - K_2\Theta_y = 0$$

$$y: \quad -K_2\Theta_o + \left(D_2 s + K_2\right)\Theta_y = 0$$

TRANSLATED TO T'S FRAME

$\gamma_{21} = r_1/r_2$

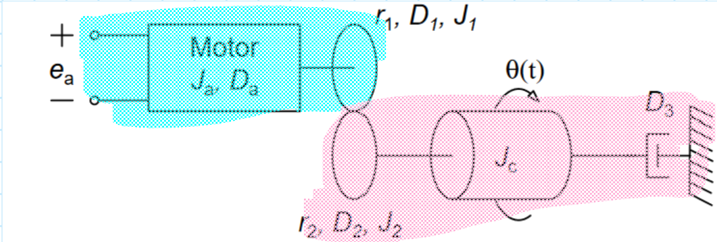


$$a: \quad \left(\left(J_1 + \gamma_{21}^2\left(J_g + M r_g^2\right)\right)s^2 + \left(D_1 + f_v r_g^2 \gamma_{21}^2\right)s + K_L r_g^2\gamma_{21}^2 + K_1\gamma_{21}^2\right)\Theta_a - K_1\gamma_{21}^2\left(-\frac{\Theta_o}{\gamma_{21}}\right) = T$$

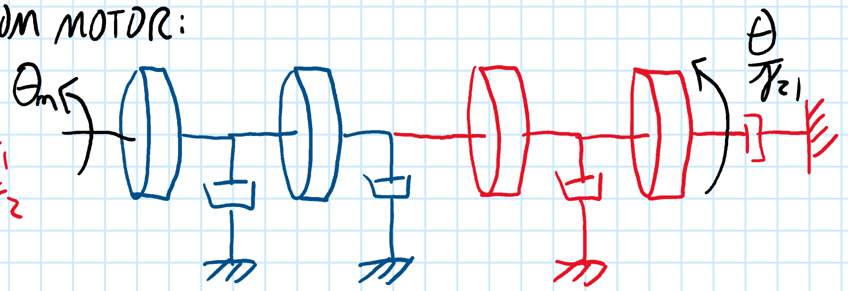
$$o: \quad -K_1\gamma_{21}^2\Theta_a + \left(J_2\gamma_{21}^2 s^2 + K_1\gamma_{21}^2 + K_2\gamma_{21}^2\right)\left(-\frac{\Theta_o}{\gamma_{21}}\right) + K_2\gamma_{21}^2\Theta_b = 0$$

$$b: \quad -K_2\gamma_{21}^2\left(-\frac{\Theta_o}{\gamma_{21}}\right) + \left(D_2\gamma_{21}^2 s + K_2\gamma_{21}^2\right)\Theta_b = 0$$

6



FROM MOTOR:



$\gamma_{21} = \frac{r_1}{r_2}$

J_a D_a J_1 D_1 $J_2\gamma_{21}^2$ $D_2\gamma_{21}^2$ $J_c\gamma_{21}^2$ $D_3\gamma_{21}^2$

$$J_m = J_a + J_1 + \gamma_{21}^2 (J_2 + J_c)$$

$$D_m = D_a + D_1 + \gamma_{21}^2 (D_2 + D_3)$$

$$\frac{\Theta_m}{E_a} = \frac{\frac{1}{J_m}\frac{K_t}{R_a}}{s\left(s + \frac{1}{J_m}\left(D_m + \frac{K_t K_b}{R_a}\right)\right)}$$

$$\theta = \gamma_{21}\theta_m$$