Duke University
Edmund T. Pratt, Jr. School of Engineering

Controls Fall 2016

Test III

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Name (please print)____

In keeping with the Community Standard, I have neither provided nor received any assistance on this test. I understand if it is later determined that I gave or received assistance, I will be brought before the Undergraduate Conduct Board and, if found responsible for academic dishonesty or academic contempt, fail the class. I also understand that I am not allowed to speak to anyone except the instructor about any aspect of this test until the instructor announces it is allowed. I understand if it is later determined that I did speak to another person about the test before the instructor said it was allowed, I will be brought before the Undergraduate Conduct Board and, if found responsible for academic dishonesty or academic contempt, fail the class.

Signature:____

Instructions

First - please turn **off** any cell phones or other annoyance-producing devices. Vibrate mode is not enough - your device needs to be in a mode where it will make no sounds during the course of the test, including the vibrate buzz or those acknowledging receipt of a text or voicemail.

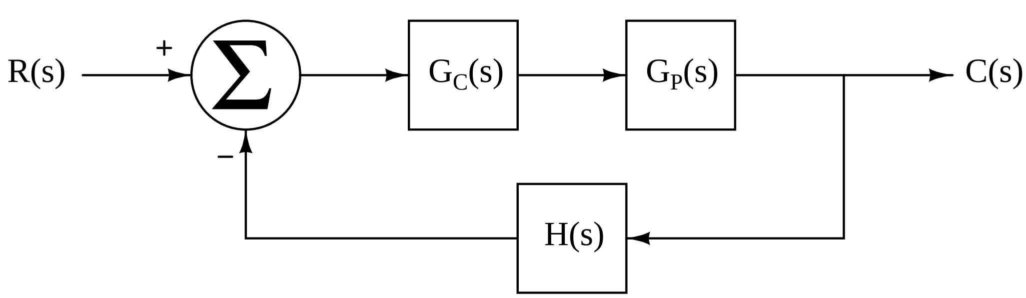
Please be sure to put each problem on its own page or pages - do *not* write answers to more than one problem on any piece of paper and do not use the back of a problem for work on a *different* problem. You will be turning in each of the problems independently. This cover page should be stapled to the front of Problem 1.

Make sure that your name *and* NetID are *clearly* written at the top of *every* page, just in case problem parts come loose in the shuffle. Make sure that the work you are submitting for an answer is clearly marked as such. Finally, when turning in the test, individually staple all the work for each problem and place each problem's work in the appropriate folder.

If you create some intermediate variable that is defined in terms of other known variables, you do *not* need to back-substitute later. For instance, if you have some $R_{eq} = R_1 + R_2$, from that point forward, you can use R_{eq} without having to expand it out.

Note that there may be people taking the test after you, so you are not allowed to talk about the test - even to people outside of this class - until I send along the OK. This includes talking about the specific problem types, how long it took you, how hard you thought it was - really anything. Please maintain the integrity of this test.

General System Model



Name (please print):
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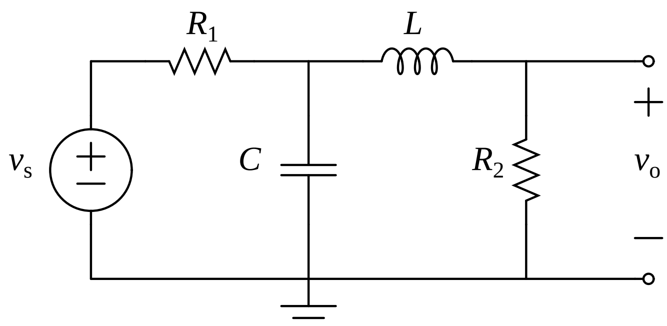
Problem I: [16 pts.] State Space

(1) Given the transfer function below for some system with an input of $x(t)$ and an output of $y(t)$, determine a set of A, B, C, D matrices as well as a state vector z which could be used to represent the same relationship between input and output. Be sure to clearly indicate what you are considering as states.

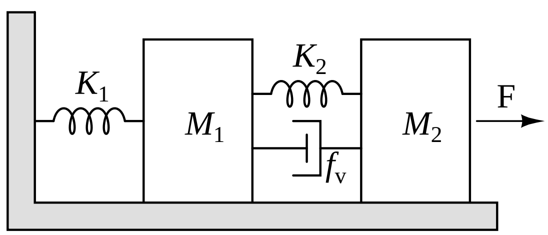
$$\mathbb{H}(s) = \frac{\mathbb{Y}(s)}{\mathbb{X}(s)} = \frac{2s+4}{4s^3+5s^2+6s+7}$$

(2) PICK EITHER (a) or (b) below - do not do both!

(a) *Either,* given the circuit below, determine a set of A, B, C, D matrices as well as a state vector z which could be used to solve for the states and for the specified output voltage. Be sure to clearly indicate what you are considering as states.



(b) *Or,* given the mechanical system below, and assuming the desired output is the position of the block M_2, determine a set of A, B, C, D matrices as well as a state vector z which could be used to solve for the states and for the specified output position. Be sure to clearly indicate what you are considering as states. The interfaces between the masses and the platform are ~~magical~~ frictionless.



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Problem II: [20 pts.] Root Loci

For the following problems, you are given a system with a gain controller $G_C = K$, a plant G_P, and an output transducer H. You are also given the overall transfer function for each system, T, which includes the gain controller. For each, you must:

(1) Determine the range of K for stability of the system.

(2) Make an accurate sketch of the root locus for positive gains K. You must clearly calculate any $j\omega$ crossings, asymptote centers, and asymptote angles. For break-in or break-out points, you should make a reasonable estimate but are not required to solve for the exact value. Neither are you required to solve for K at the break-in or break-out points.

(3) Determine the locations of any closed-loop zeros for the system.

(4) Determine the system type(s) for each, the appropriate finite static error constant(s), and the appropriate steady state error in response to $u(t)$, $tu(t)$, and/or $\frac{1}{2}t^2u(t)$. *Be careful! And complete!*

$$G_{P1} = \frac{1}{s(s+2)(s+4)} \qquad H_1 = 1 \qquad T_1 = \frac{K}{s^3+6s^2+8s+K} \tag{1}$$

$$G_{P2} = \frac{1}{(s-2)(s+3)} \qquad H_2 = \frac{s+5}{s+4} \qquad T_2 = \frac{K(s+4)}{s^3+5s^2+(K-2)s+(5K-24)} \tag{2}$$

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Problem III: [28 pts.] System Categorization and Control I

Assume a unity feedback system with input $r(t)$, output $c(t)$, a gain controller $G_{C1} = K$, and a plant with a transfer function equal to:

$$G_P = \frac{1}{(s^2+8s+25)(s^2+2s+10)} = \frac{1}{((s+4)^2+(3)^2)((s+1)^2+(3)^2)} = \frac{1}{s^4+10s^3+51s^2+130s+250}$$

Note that a break-in/break-out point analysis reveals that

$$\frac{dK(\sigma)}{d\sigma} = 0 \text{ when } \sigma = -2.5$$

while calculating the overall transfer function with a gain controller $G_{C1} = K$ yields:

$$T = \frac{K}{s^4+10s^3+51s^2+130s+250+K}$$

Furthermore, creating a Routh Array of the overall transfer function yields:

$$\begin{bmatrix} 1 & 51 & K+250 & s^4 \\ 1 & 13 & 0 & s^3 \\ 38 & K+250 & 0 & s^2 \\ 244-K & 0 & 0 & s \\ K+250 & 0 & 0 & 1 \end{bmatrix}$$

(1) What is the range of stability for a gain controller? Include both positive and negative values if they are stable.

(2) Sketch an accurate representation of the root locus plot for this system with gain controller G_{C1} for positive gains. You must clearly calculate any $j\omega$ crossings, asymptote centers, asymptote angles, and break-in/break-out locations.

(3) Can this system's dominant poles ever be critically damped with positive gain? If not, state why you believe that. If so, determine the location of the dominant poles and the value for the gain to make the system's dominant poles critically damped with positive gain.

(4) Assuming positive gain, what is the shortest settling time for this system? Why do you believe that?

(5) Determine the system type, the value of the appropriate finite, non-zero valued static error constant, and the value of the finite, non-zero valued steady state error relative to a unit input. What is the function of the input in this case?

(6) Assuming $G_{C1} = 100$, SISOtool indicates that the closed loop poles are at:

$$s = -0.463 \pm j3.3, -4.54 \pm j3.3$$

(a) Which poles are dominant?

(b) What are the predicted %OS and T_s for those dominant pole locations?

(c) How likely is it that a simulation of the system would demonstrate those characteristics? Why do you believe that?

(d) What will the steady-state error be if $G_{C1} = 100$?

(e) Design a passive controller G_{C2} which will reduce the steady-state error in the system above by a factor of 2 without having a major impact on the transient response. Clearly indicate every step of determining any poles, zeros, or gains in your controller and provide reasons for any decisions you have to make about those values. What is this kind of controller called?

(f) Remove G_{C2} and instead design an active controller G_{C3} which will increase the system type by 1 without having a major impact on the transient response. Clearly indicate every step of determining any poles, zeros, or gains in your controller and provide reasons for any decisions you have to make about those values. What is this kind of controller called?

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Problem IV: [36 pts.] System Categorization and Control II

Assume a unity feedback system with input $r(t)$, output $c(t)$, a gain controller $G_{C1} = K$, and a plant with a transfer function:

$$G_P = \frac{1}{(s+3)(s+7)} = \frac{1}{s^2 + 10s + 21}$$

Note that a break-in/break-out point analysis reveals that

$$\frac{dK(\sigma)}{d\sigma} = 0 \text{ when } \sigma = -5$$

(1) What is the range of stability for a gain controller? Include both positive and negative values if they are stable.

(2) Sketch an accurate representation of the root locus plot for this system with gain controller G_{C1} for positive gains. You must clearly calculate any $j\omega$ crossings, asymptote centers, asymptote angles, and break-in/break-out locations.

(3) Can this system's dominant poles ever be critically damped with positive gain? If not, state why you believe that. If so, determine the location of the dominant poles and the value for the gain to make the system's dominant poles critically damped.

(4) What is the shortest settling time for this system? Why do you believe that?

(5) Determine the pole location, using gain control, that obtains that fastest settling time while simultaneously having a damping ratio of 0.9. Also determine the value of the gain K that puts the poles at the appropriate locations.

(6) What would the dominant pole locations be for a system operating with a faster-than-possible-with-gain-control-for-this-system settling time of 0.5 s and a damping ratio of 0.9?

(7) Replace the gain controller $G_{C1} = K$ with a PD controller G_{C2} meant to obtain dominant system poles such that the system has a settling time of 0.5 s and a damping ratio of 0.9. Clearly indicate every step of determining any poles, zeros, or gains in your controller and provide reasons for any decisions you have to make about those values.

(8) Replace the PD controller with a passive controller G_{C3} meant to obtain dominant system poles such that the system has a settling time of 0.5 s and a damping ratio of 0.9 and also which should perfectly follow the second-order assumptions. Clearly indicate every step of determining any poles, zeros, or gains in your controller and provide reasons for any decisions you have to make about those values. What is this kind of controller called?

(9) Assuming G_{C3} is in place, determine the system type, the value of the appropriate finite, non-zero valued static error constant, and the value of the finite, non-zero valued steady state error relative to a unit input. What is the function of the input in this case?