Duke University
Edmund T. Pratt, Jr. School of Engineering

Controls Fall 2016

System Model Quiz

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Name and NetID (please print) ______________________________

In keeping with the Community Standard, I have neither provided nor received any assistance on this quiz. I understand if it is later determined that I gave or received assistance, I will be brought before the Undergraduate Conduct Board and, if found responsible for academic dishonesty or academic contempt, fail the class. I also understand that I am not allowed to speak to anyone except the instructor about any aspect of this quiz until the instructor announces it is allowed. I understand if it is later determined that I did speak to another person about the quiz before the instructor said it was allowed, I will be brought before the Undergraduate Conduct Board and, if found responsible for academic dishonesty or academic contempt, fail the class.

Signature:______________________________

Instructions

First - please turn **off** any cell phones or other annoyance-producing devices. Vibrate mode is not enough - your device needs to be in a mode where it will make no sounds during the course of the quiz, including the vibrate buzz or those acknowledging receipt of a text or voicemail.

Please be sure to put each problem on its own page or pages - do *not* write answers to more than one problem on any piece of paper and do not use the back of a problem for work on a *different* problem. You will be turning in each of the problems independently. This cover page should be stapled to the front of Problem 1.

Make sure that your name *and* NetID are *clearly* written at the top of *every* page, just in case problem parts come loose in the shuffle. Make sure that the work you are submitting for an answer is clearly marked as such. Finally, when turning in the test, individually staple all the work for each problem and place each problem's work in the appropriate folder.

If you create some intermediate variable that is defined in terms of other known variables, you do *not* need to back-substitute later. For instance, if you have some $R_{\text{eq}} = R_1 + R_2$, from that point forward, you can use R_{eq} without having to expand it out.

Note that there may be people taking the quiz after you, so you are not allowed to talk about the quiz - even to people outside of this class - until I send along the OK. This includes talking about the specific problem types, how long it took you, how hard you thought it was - really anything. Please maintain the integrity of this quiz.

Equations

Motor Equation

$$\frac{\Theta_{\text{m}}(s)}{\mathbb{E}_{\text{a}}(s)} = \frac{\frac{1}{J_{\text{m}}}\frac{K_{\text{t}}}{R_{\text{a}}}}{s\left(s + \frac{1}{J_{\text{m}}}\left(D_{\text{m}} + \frac{K_{\text{t}}K_{\text{b}}}{R_{\text{a}}}\right)\right)}$$

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Problem I: [26 pts.] Math Tools

(1) *Clearly* using Laplace techniques, determine the output $x(t)$ for $t > 0$ for a system which can be modeled as:

$$\frac{dx^2(t)}{dt^2} + 8\frac{dx(t)}{dt} + 7x(t) = e^{-2t}u(t)$$
$$x(0^-) = 3$$
$$\dot{x}(0^-) = 1$$

(2) Determine the impulse response $h(t)$ for a system with the following transfer function:

$$\mathbb{H}(s) = \frac{5s + 13}{s^2 + 8s + 20}$$

(3) A linear system with input $r(t)$ and output $c(t)$ has a step response of:

$$s_r(t) = (5 - 5e^{-3t}\cos(6t))u(t)$$

(a) Determine the transfer function for the system $\mathbb{G} = \frac{\mathbb{C}}{\mathbb{R}}$, and
(b) Determine the impulse response of the system.

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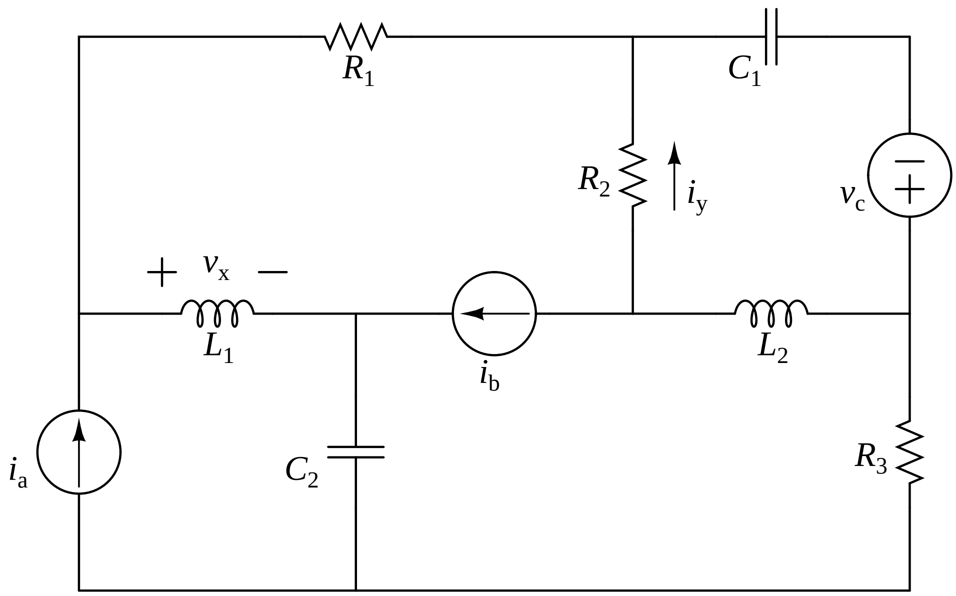
Problem II: [20 pts.] Electrical Systems

Given the following electrical system:



and assuming that the values for the passive elements (L_1, L_2, R_1, R_2, R_3 C_1, C_2), and the values for the independent sources (i_a, i_b, v_c) are known, *clearly* demonstrate the use of the Mesh Current Method in the frequency domain to label unknowns for the circuit and to determine a complete set of equations in the frequency domain that could be used to solve for these unknowns, including the voltage measurement $\mathbb{V}_x$ and the current measurement $\mathbb{I}_y$. List the set of unknowns you believe your equations will find. You do not need to arrange the equations in matrix format nor do you need to solve the equations.

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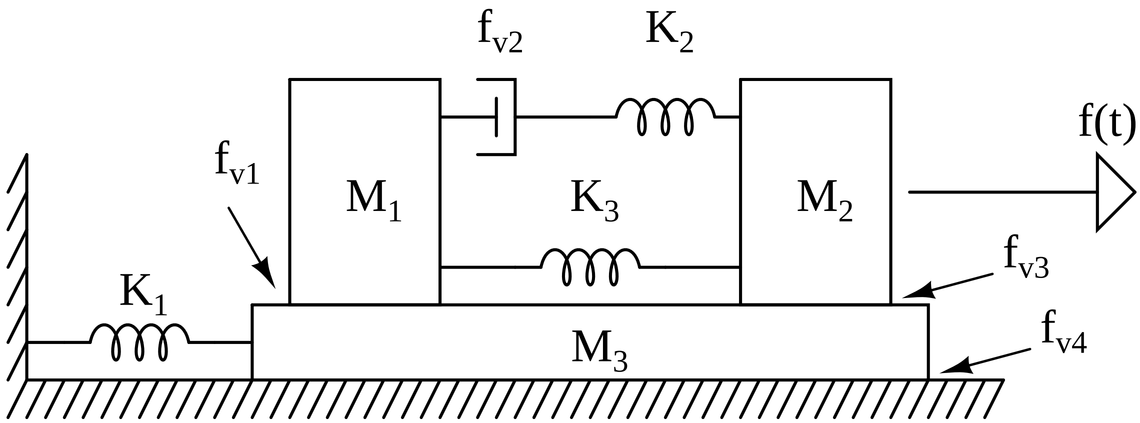
Problem III: [18 pts.] Translational Systems

Given the following system:



where $f(t)$ is a known force applied to mass M_2 and assuming that the values of the passive elements are known, *clearly* determine a complete set of equations of motion for the system in the frequency domain. List the set of unknowns you believe your equations use and be sure they are all clearly labeled on the diagram. You do not need to arrange the equations in matrix format nor do you need to solve the equations.

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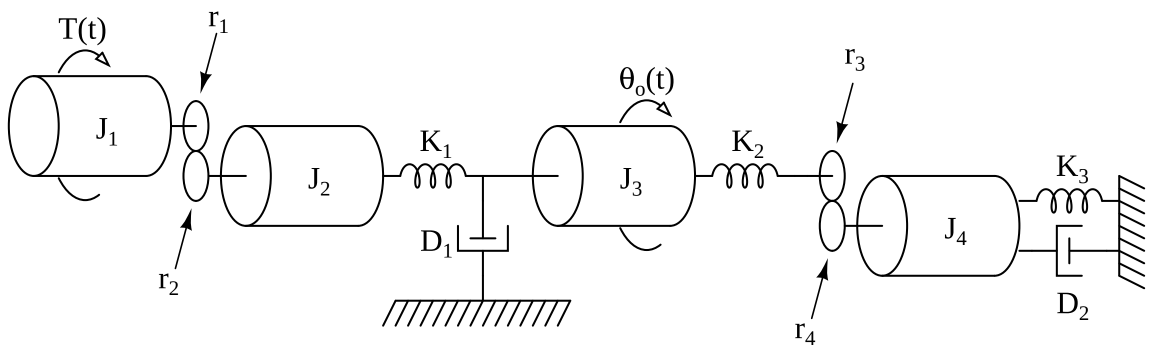
Problem IV: [18 pts.] Rotational Systems

Given the following system:



where $T(t)$ is a torque applied to inertia J_1 and the output is the angle $\theta_o(t)$ through which inertia J_3 rotates, and assuming that the values of the passive elements, the torque, and the gear radii are known, *clearly* determine the equations of motion for the system in the frequency domain. List the set of unknowns you believe your equations use and be sure they are all clearly labeled on the diagram. $\mathbf{\Theta_o(s)}$ must be one of the unknowns for which your equations would solve. If you choose to introduce any additional measurements or constants, be sure to clearly define them on your diagram. You do not need to arrange the equations in matrix format nor do you need to solve the equations. You may assume the gears are ideal and are both undamped and inertialess.

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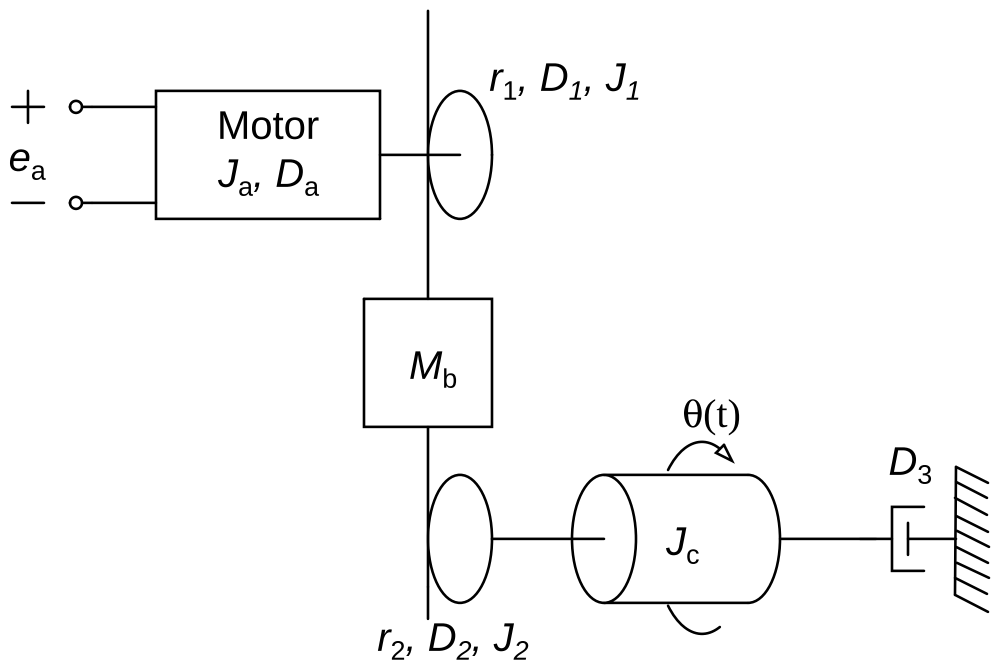
Problem V: [18 pts.] Motor Systems

Given the following system:



and assuming that the values of the passive elements, the physical properties of the motor J_a and D_a, the motor constants (K_t, K_b, R_a), and the gear radii are known,

(1) Redraw the system from the perspective of the motor, including labels, and then

(2) *Clearly* determine the transfer function

$$\mathbb{H}(s) = \frac{\mathbf{\Theta}(s)}{\mathbb{E}_a(s)}$$

in terms of the element values, motor constants, and gear radii. You may assume that a positive voltage applied to the motor will result in a positive change in $\theta(t)$. Note that all the gears have their own inertia and damping. Also, the rack to which M_b is attached is behind both pinion gears.