

Controls Test II F2016

Note Title

$$I) (1) \quad T = \frac{10}{s+4} \quad c_{ss} = \lim_{s \rightarrow 0} s \cdot \frac{10}{s(s+4)} = 2.5 \quad T_s = \frac{4}{4} = 1 \text{ sec}$$

$$(2) \quad T = \frac{\frac{15}{s+5}}{1 + \frac{15}{s+5}} = \frac{15}{s+20} \quad c_{ss} = \lim_{s \rightarrow 0} \frac{s \cdot 15}{s(s+20)} = 0.75 \quad T_s = \frac{4}{20} = 0.2 \text{ sec}$$

$$(3) \quad T = \frac{12}{s^2+4s+20} \quad \omega_n = \sqrt{20} = 2\sqrt{5} = 4.472 \quad T_s \approx \frac{4}{\zeta \omega_n} = 2 \text{ sec}$$

$$c_{ss} = \frac{12}{20} = \frac{3}{5} \quad 2\zeta \omega_n = 4 \quad \zeta = \frac{4}{2\omega_n} = \frac{1}{\sqrt{5}} = 0.447 \quad \%OS = \exp\left(\frac{-\zeta\pi}{\sqrt{1-\zeta^2}}\right) \times 100 = 20.8\%$$

$$(4) \quad T = \frac{25}{s^2+6s+45} \quad \omega_n = \sqrt{45} = 6.708 \quad T_s \approx \frac{4}{\zeta \omega_n} = 1.33 \text{ sec}$$

$$c_{ss} = \frac{25}{45} = \frac{5}{9} \quad 2\zeta \omega_n = 6 \quad \zeta = \frac{6}{2\omega_n} = \frac{3}{\sqrt{45}} = 0.359 \quad \%OS = \exp\left(\frac{-\zeta\pi}{\sqrt{1-\zeta^2}}\right) \times 100 = 29.9\%$$

$$(5) \quad c_{ss} = 11 \quad T = \frac{(11)\left(\frac{2}{3}\right)}{s + \frac{2}{3}} = \frac{22}{3s+2}$$

$$T_s = 6 \quad \text{so} \quad a = \frac{4}{6} = \frac{2}{3}$$

$$(6) \quad c_{ss} = 1/4 \quad \omega_d = \frac{\pi}{T_p} = \frac{\pi}{3} \quad \omega_n = \frac{\omega_d}{\sqrt{1-\zeta^2}} = 1.072$$

$$\%OS = 50 \quad \zeta = \frac{-\ln(\%OS/100)}{\sqrt{\pi^2 + \ln^2(\%OS/100)}} = 0.215$$

$$T_p = 3$$

$$T = \frac{\left(\frac{1}{4}\right)(1.072)^2}{s^2 + 2(0.215 \times 1.072)s + (1.072)^2} = \frac{0.288}{s^2 + 0.461s + 1.15}$$

$$\begin{array}{ll}
 \text{II) } T_1: & G_1 G_2 G_4 & L_1: -G_2 G_3 \\
 T_2: & G_1 G_2 G_8 & L_2: -G_4 G_5 \\
 T_3: & G_7 G_8 & L_3: -G_2 G_4 G_6 \\
 & & L_4: -G_2 G_8 G_6 \\
 & & L_5: -G_8 G_9
 \end{array}$$

$$\Delta = 1 - L_1 - L_2 - L_3 - L_4 - L_5 + L_1 L_2 + L_1 L_5 + L_2 L_4 + L_2 L_5 + L_3 L_5 - L_1 L_2 L_5$$

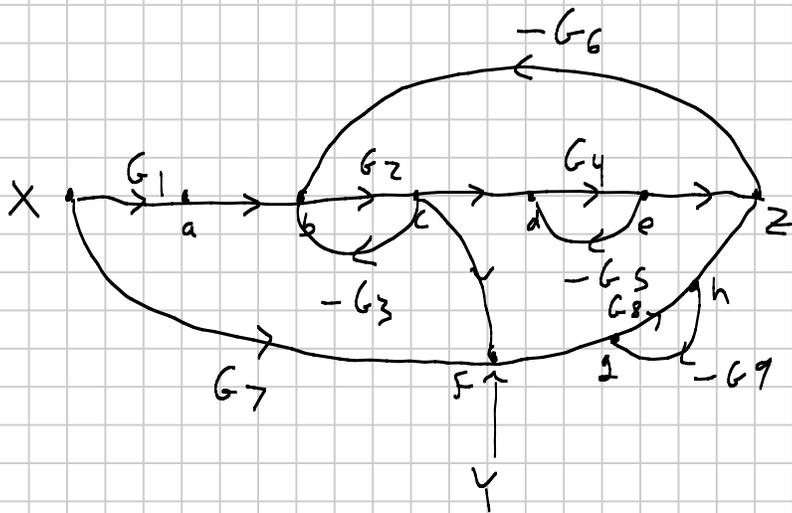
$$\Delta_1 = 1 - L_5 \quad \Delta_2 = 1 - L_2 \quad \Delta_3 = 1 - L_1 - L_2 + L_1 L_2$$

$$T = \frac{\sum T_k \Delta_k}{\Delta}$$

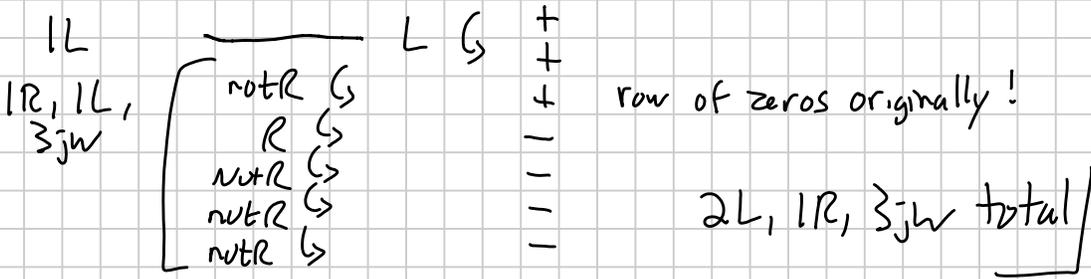
$$\begin{array}{ll}
 T_1: & G_8 & L_1: -G_2 G_3 \\
 & & L_2: -G_4 G_5 \\
 & & L_3: -G_2 G_4 G_6 \\
 & & L_4: -G_2 G_8 G_6 \\
 & & L_5: -G_8 G_9
 \end{array}$$

$\Delta = \text{same as above}$

$$\Delta_1 = 1 - L_1 - L_2 + L_1 L_2$$



III) (1) $T = \frac{600(s+1)}{s^6 + 2s^5 - 7s^4 - 14s^3 - 144s^2 - 288s}$



(2) $G = \frac{10}{s^3 + 10s + 15}$ $T = \frac{10}{s^3 + 10s + 25}$

s^3	1	10	if $\epsilon > 0$,
s^2	$\emptyset \epsilon$	25	+
s^1	$\frac{25 - 10\epsilon}{-\epsilon}$		+
s^0	25		-
			+

L
R
R

2R, 1L

(3) $T = \frac{G}{1+GH} = \frac{50}{s^2 + 7s + 12} \cdot \frac{4a}{s^2 + 7s + 12 + \frac{4a}{s+a}} = \frac{50(s+a)}{s^3 + 7s^2 + 12s + as^2 + 7as + 12a + 200a}$

s^3	1	$12 + 7a$	
s^2	$7+a$	$212a$	$a > -7$
s^1	$\frac{212a - 7a^2 - 61a - 84}{-(7+a)}$		if $a > -7$, $-7a^2 + 151a - 84$
s^0	$212a$		$a > 0$

$7a^2 - 151a + 84 > 0$

roots are $a = \frac{151 \pm \sqrt{151^2 - 4 \cdot 7 \cdot 84}}{2 \cdot 7} = \frac{151 \pm 143}{14}$
 $= \frac{4}{7}, 21$

so, either $a > 4/7$ and $a > 21$ or
 $a < 4/7$ and $a < 21$

Put together: $0 < a < 4/7$ or $a > 21$

b) $a = 4/7$ gives $(7 + 4/7)s^2 + 212 \cdot 4/7 = 0, w = 4$ Note: $a = 0$ means $\omega = 0$ not marginal
 $a = 21$ gives $(7 + 21)s^2 + 212 \cdot 21 = 0, w = \sqrt{159} = 12.61$

c) $G_{eg} = \frac{T}{1-T} = \frac{50(s+a)}{s^3 + 7s^2 - 38s + as^2 + 7as + 162a}$ all stable a , Type 0
 $K_p = 50a/162a = 0.309, e = 1/w_{np} = 0.764$

IV)

$$G_{og} = \frac{50K}{s^3 + 9s^2 + 20s} \quad T = \frac{G_{og}}{1 + G_{og}} = \frac{50K}{s^3 + 9s^2 + 20s + 50K}$$

(1) Type 1

(2) K_p is ∞ K_v is neither K_a is 0

$$(3) K_v = \lim_{s \rightarrow 0} s G_{og} = \frac{5K}{2}$$

$$(4) e_{ss} = \frac{1}{K_v} = \frac{2}{5K} = 0.3 \text{ so } K = \frac{2}{0.3 \cdot 5} = \frac{4}{3}$$

if steady state position error of 30% is 4,

$$\text{slope must be } \frac{4}{.3} = 13.\bar{3} \text{ so } 13.\bar{3} \text{ t u(t)}$$

$$(5) \begin{array}{l} s^3 \quad 1 \quad 20 \\ s^2 \quad 9 \quad 50K \\ s^1 \quad \frac{50K - 180}{-9} \\ s^0 \quad 50K \end{array}$$

$$K < 18/5 \quad \text{max } K = 18/5$$

$$K > 0 \quad \text{min } e_{ss} = \frac{2}{5 \cdot K} = 11.1\%$$

so for $13.\bar{3}$ t u(t) min. error is 1.48

(6) Dominant poles at $-0.157 \pm j 4.153 = -\zeta \omega_n \pm j \omega_n \sqrt{1 - \zeta^2}$

$$T_s = \frac{4}{.157} = 25.5 \text{ sec}$$

$$\exp\left(\frac{-\zeta \pi}{\sqrt{1 - \zeta^2}}\right) = \exp\left(\frac{-\zeta \omega_n \pi}{\omega_n \sqrt{1 - \zeta^2}}\right) = \exp\left(\frac{-0.157 \pi}{4.153}\right) = 88.8\%$$

Very confident since there are no

zeros and other pole is 55x further from origin.

$$V) T = \frac{K(s+3)}{s^2+3s+Ks+2+3K} \quad G_{eg} = \frac{T}{1-T} = \frac{K(s+3)}{s^2+3s+2}$$

(1) Type 0

(2) K_p is neither, $K_v = K_a$ are 0

$$(3) K_p = \lim_{s \rightarrow 0} G_{eg} = \frac{3K}{2}$$

$$(4) e_{ss} = \frac{1}{1+K_p} = \frac{1}{1+\frac{3}{2}K} = 0.3, \quad K = \left(\frac{1}{0.3} - 1\right) \frac{2}{3} = 1.5$$

$$\frac{4}{0.3} = 13.3 \text{ u(t)}$$

(5) denom. of T is $s^2 + (3+K)s + (2+3K)$ so.

for stability, so $K+3 > 0$ $2+3K > 0$
 $K > -3$ $K > -2/3$

$$\omega_n = \sqrt{2+3K} \quad 2\zeta\omega_n = 3+K \quad \text{so } \zeta = \frac{3+K}{2\sqrt{2+3K}}$$

underdamped if $0 < \zeta < 1$ so

$$0 < \frac{3+K}{2\sqrt{2+3K}} < 1$$

$\frac{3+K}{2\sqrt{2+3K}} = 0$ if
 $K = -3$ but
 unstable.

$\frac{3+K}{2\sqrt{2+3K}} < 1$ $3+K < 2\sqrt{2+3K}$
 if $K > -2/3$, can square both sides

$$K^2 + 6K + 9 < 12K + 8 \quad \text{so}$$

$$K^2 - 6K + 1 < 0$$

$$K = \frac{6 \pm \sqrt{36-4}}{2} = 3 \pm 2\sqrt{2} = 0.172, 5.83$$

$$(K - 0.172)(K - 5.83) < 0$$

so For underdamped, $0.172 < K < 5.83$

(6) (a) To get $s = -2 \pm j$, denom of $(s+2)^2 + 1 = s^2 + 4s + 5$
 means $K = 1$

$$(b) -2 \pm j = -\zeta\omega_n \pm j\omega_n\sqrt{1-\zeta^2}$$

$$T_s = \frac{4}{\zeta} = 2 \text{ sec} \quad \%OS = \exp\left(\frac{-2\pi}{1}\right) \times 100\% = 0.187\%$$

(c) There is a zero at -3 so ... not confident
 (Simulated $T_s = 1.5s$, $\%OS = 0.432\%$)