

Duke University  
Edmund T. Pratt, Jr. School of Engineering

Controls Fall 2015  
Test III  
Michael R. Gustafson II

---

Name (please print) \_\_\_\_\_

In keeping with the Community Standard, I have neither provided nor received any assistance on this test. I understand if it is later determined that I gave or received assistance, I will be brought before the Undergraduate Conduct Board and, if found responsible for academic dishonesty or academic contempt, fail the class. I also understand that I am not allowed to speak to anyone except the instructor about any aspect of this test until the instructor announces it is allowed. I understand if it is later determined that I did speak to another person about the test before the instructor said it was allowed, I will be brought before the Undergraduate Conduct Board and, if found responsible for academic dishonesty or academic contempt, fail the class.

Signature: \_\_\_\_\_

---

## Instructions

First - please turn **off** any cell phones or other annoyance-producing devices. Vibrate mode is not enough - your device needs to be in a mode where it will make no sounds during the course of the test, including the vibrate buzz or those acknowledging receipt of a text or voicemail.

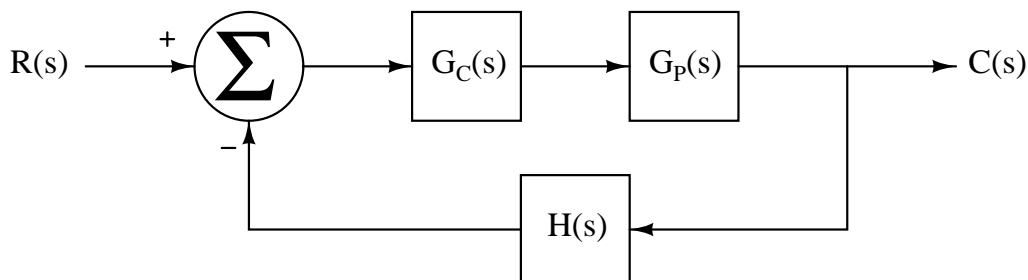
Please be sure to put each problem on its own page or pages - do *not* write answers to more than one problem on any piece of paper and do not use the back of a problem for work on a *different* problem. You will be turning in each of the problems independently. This cover page should be stapled to the front of Problem 1.

Make sure that your name *and* NetID are *clearly* written at the top of *every* page, just in case problem parts come loose in the shuffle. Make sure that the work you are submitting for an answer is clearly marked as such. Finally, when turning in the test, individually staple all the work for each problem and place each problem's work in the appropriate folder.

If you create some intermediate variable that is defined in terms of other known variables, you do *not* need to back-substitute later. For instance, if you have some  $R_{eq} = R_1 + R_2$ , from that point forward, you can use  $R_{eq}$  without having to expand it out.

Note that there may be people taking the test after you, so you are not allowed to talk about the test - even to people outside of this class - until I send along the OK. This includes talking about the specific problem types, how long it took you, how hard you thought it was - really anything. Please maintain the integrity of this test.

## General System Model



Name (please print):

Community Standard (print NetID):

### Problem I: [40 pts.] General Root Loci

For the following problems, you are given a system with a gain controller  $G_C = K$ , a plant  $G_P$ , and an output transducer  $H$ . For each, you must:

- (a) Determine the range of  $K$  for stability of the system.
- (b) Make an accurate sketch the root locus for positive gains  $K$ . You must clearly calculate any  $j\omega$  crossings, asymptote centers, and asymptote angles. For break-in or break-out points, you should make a reasonable estimate but are not required to solve for the exact value. Neither are you required to solve for  $K$  at the break-in or break-out points.
- (c) Determine the location of the closed-loop zeros for the system.
- (d) Determine the system type for each, the appropriate finite static error constant, and the appropriate steady state error in response to either  $u(t)$ ,  $tu(t)$ , or  $\frac{1}{2}t^2u(t)$ .

$$G_{P1} = \frac{1}{(s)(s+6)} \qquad H_1 = 1 \qquad (1)$$

$$G_{P2} = \frac{s+4}{(s-1)(s+2)} \qquad H_2 = 1 \qquad (2)$$

$$G_{P3} = \frac{1}{(s-1)(s+4)(s+9)} \qquad H_3 = 1 \qquad (3)$$

$$G_{P4} = \frac{1}{(s)(s+3)} \qquad H_4 = \frac{s+2}{s+1} \qquad (4)$$

Name (please print):

Community Standard (print NetID):

## Problem II: [30 pts.] System Categorization and Control I

Assume a unity feedback system with input  $r(t)$ , output  $c(t)$ , a gain controller  $G_{C1} = K$ , and a plant with a transfer function equal to that of  $G_{P3}$  of the first problem - that is,

$$G_P = \frac{1}{(s-1)(s+4)(s+9)}$$

Note that a break-in/break-out point analysis reveals that

$$\frac{dK(\sigma)}{d\sigma} = 0 \text{ when } \sigma = -6.89, -1.11$$

- (1) Can this system's dominant poles ever be critically damped with positive gain? If not, state why you believe that. If so, determine the location of the dominant poles and the value for the gain to make the system's dominant poles critically damped with positive gain.
- (2) What is the shortest settling time for this system? Why do you believe that?
- (3) Determine the dominant system pole locations that would describe a system that has a 10% overshoot and a settling time of 1.5 seconds.
- (4) Since proportional control alone will not be able to achieve these pole locations, replace the gain controller  $G_{C1} = K$  with an active controller  $G_{C2}$  that *will* allow this system to achieve the transient characteristics of 10% overshoot and a settling time of 1.5 seconds. Clearly indicate every step of determining any poles, zeros, or gains in your controller. What is this kind of controller called?
- (5) Replace the active controller  $G_{C2}$  with a passive controller  $G_{C3}$  that will allow this system to achieve the transient characteristics of 10% overshoot and a settling time of 1.5 seconds and is *highly likely* to have its transient characteristics match those calculated using second-order approximations. Clearly indicate every step of determining any poles, zeros, or gains in your controller. What is this kind of controller called?

Name (please print):

Community Standard (print NetID):

### Problem III: [30 pts.] System Categorization and Control II

Assume a unity feedback system with input  $r(t)$ , output  $c(t)$ , a gain controller  $G_{C1} = K$ , and a plant with a transfer function equal to that of  $G_{P1}$  of the first problem - that is,

$$G_P = \frac{1}{(s)(s+6)}$$

Note that a break-in/break-out point analysis reveals that

$$\frac{dK(\sigma)}{d\sigma} = 0 \text{ when } \sigma = -3$$

- (1) Can this system's dominant poles ever be critically damped with positive gain? If not, state why you believe that. If so, determine the location of the dominant poles and the value for the gain to make the system's dominant poles critically damped.
- (2) What is the shortest settling time for this system? Why do you believe that?
- (3) Determine the pole location, using gain control, that obtains that fastest settling time while simultaneously having 20% overshoot. Also determine the value of the gain  $K$  that puts the poles at the appropriate locations.
- (4) Replace the gain controller  $G_{C1} = K$  with a *passive* controller  $G_{C2}$  meant to keep the second order poles at the same location as above but that decreases the finite steady state error by a factor of three relative to what you found for this system in Problem I. Clearly indicate every step of determining any poles, zeros, or gains in your controller and provide reasons for any decisions you have to make about those values. What is this kind of controller called?
- (5) Assuming  $G_{C2}$  is in place, determine the system type, the value of the appropriate finite, non-zero valued static error constant, and the value of the finite, non-zero valued steady state error relative to a unit input. What is the function of the input in this case?
- (6) Replace the controller  $G_{C2}$  with an *active* controller  $G_{C3}$  meant to keep the second order poles at the same location as above but that raises the system type. Clearly indicate every step of determining any poles, zeros, or gains in your controller and provide reasons for any decisions you have to make about those values. What is this kind of controller called?
- (7) Assuming  $G_{C3}$  is in place, determine the system type, the value of the appropriate finite, non-zero valued static error constant, and the value of the finite, non-zero valued steady state error relative to a unit input. What is the function of the input in this case?