

Test I Fall 2015 Controls

Note Title

I (1)

$$\ddot{x} + 5\dot{x} + 6x = e^{-t}u(t)$$

$$(s^2X - sX(0^-) - \dot{x}(0^-)) + S(sX - x(0^-)) + 6X = \frac{1}{s+1}$$

$$(s^2 + 5s + 6)X = sX(0^-) + \dot{x}(0^-) + Sx(0^-) + \frac{1}{s+1}$$

$$4s + 20 + \frac{1}{s+1} = \frac{4s^2 + 24s + 21}{s+1}$$

$$X = \frac{4s^2 + 24s + 21}{(s^2 + 5s + 6)(s+1)} = \frac{4s^2 + 24s + 21}{(s+1)(s+2)(s+3)} = \frac{A}{s+1} + \frac{B}{s+2} + \frac{C}{s+3}$$

$$A = \lim_{s \rightarrow -1} \frac{4s^2 + 24s + 21}{(s+2)(s+3)} = \frac{4 - 24 + 21}{(-1)(2)} = \frac{1}{2}$$

$$B = \lim_{s \rightarrow -2} \frac{4s^2 + 24s + 21}{(s+1)(s+3)} = \frac{16 - 48 + 21}{(-1)(1)} = 11$$

$$C = \lim_{s \rightarrow -3} \frac{4s^2 + 24s + 21}{(s+1)(s+2)} = \frac{36 - 72 + 21}{(-2)(-1)} = -\frac{15}{2}$$

$$x(t) = 0.5e^{-t} + 11e^{-2t} - 7.5e^{-3t}$$

$$\text{check: } x(0) = \frac{1}{2} + \frac{11}{2} - \frac{15}{2} = \frac{8}{2} = 4 \quad \checkmark$$

$$I (2) \quad h(t) = \mathcal{L}^{-1}\{H(s)\} = \mathcal{L}^{-1}\left\{\frac{20s+10}{s^2+4s+13}\right\} \rightarrow \frac{20s+10}{(s+\alpha)^2 + (\beta)^2}$$

$\uparrow b^2 - 4ac < 0, \text{ MOAT}$
complete the square

$$\frac{20s+10}{(s+2)^2 + (3)^2} = \frac{A(s+2) + B(3)}{(s+2)^2 + (3)^2} \quad A=20 \quad 40 + B \cdot 3 = 10 \\ B=-10$$

$$h(t) = e^{-2t} (20 \cos(3t) - 10 \sin(3t))$$

$$I (3) \quad R(s) = \frac{s}{s^2 + 1^2} \quad C(s) = \frac{8s}{s^2 + 1^2} + \frac{2(1)}{s^2 + 1^2} - \frac{8}{s+4}$$

$$C(s) = \frac{(8s+2)(s+4) - (8)(s^2 + 1^2)}{(s^2 + 1^2)(s+4)}$$

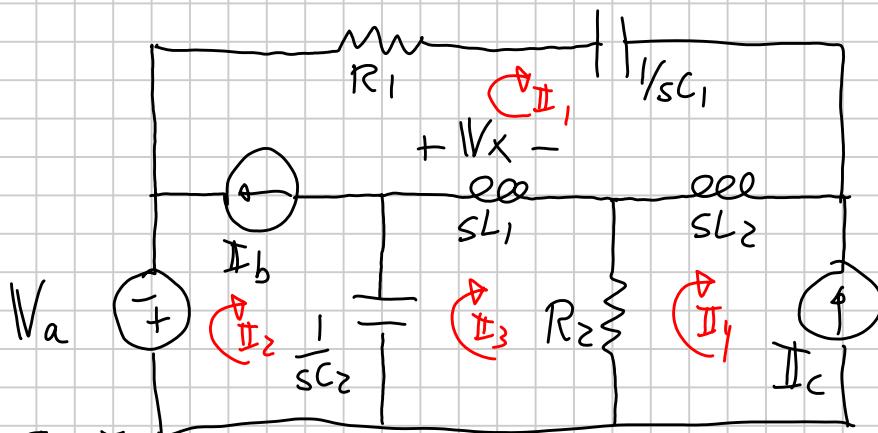
$$= \frac{8s^2 + 34s + 8 - 8s^2 - 8}{(s^2 + 1^2)(s+4)} = \frac{34s}{(s^2 + 1^2)(s+4)}$$

$$G = \frac{C}{R} = \frac{34}{s+4}$$

$$\mathcal{L}^{-1}\left\{\frac{34}{s(s+4)}\right\} = \mathcal{L}^{-1}\left\{\frac{8s}{s} - \frac{8s}{s+4}\right\}$$

$$= 8s - 8s e^{-4t}$$

II)



Unknowns: I_1, I_2, I_3, I_4, V_x

$$KVL, I_3: \frac{1}{SC_2} (I_3 - I_2) + SL_1 (I_3 - I_1) + R_2 (I_3 - I_4) = 0$$

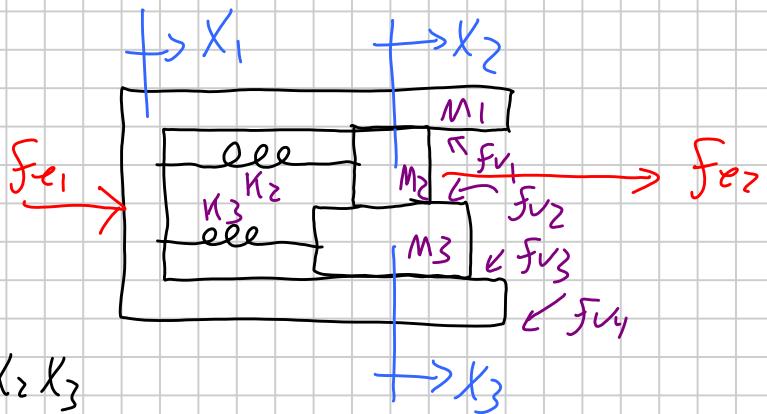
$$KVL, SL_1: V_a + R_1 I_1 + \frac{1}{SC_1} I_1 + SL_2 (I_1 - I_4) + SL_1 (I_1 - I_3) + \frac{1}{SC_2} (I_2 - I_3) = 0$$

$$SRC I_b: I_b = I_1 - I_2$$

$$SRC I_c: I_c = -I_4$$

$$MEAS: V_x = SL_1 (I_3 - I_1)$$

III)



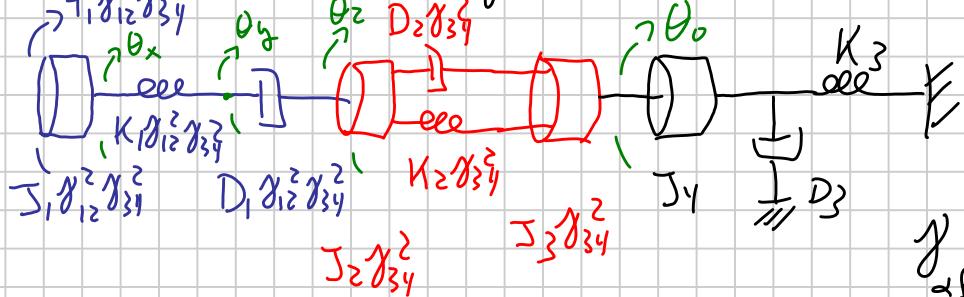
Unk. $X_1 X_2 X_3$

$$M_1: (M_1 s^2 + (f_{v1} + f_{v3} + f_{v4})s + K_2 + K_3)X_1 - (f_{v1}s + K_2)X_2 - (f_{v3}s + K_3)X_3 = F_{e1}$$

$$M_2: -(f_{v1}s + K_2)X_1 + (M_2 s^2 + (f_{v1} + f_{v2})s + K_2)X_2 - (f_{v2}s)X_3 = F_{e2}$$

$$M_3: -(f_{v2}s)X_1 - (f_{v2}s)X_2 + (M_3 s^2 + (f_{v2} + f_{v3})s + K_3)X_3 = 0$$

IV) Since θ_o is main required output, redraw from its perspective



$$\gamma_{\alpha\beta} = \frac{r_B}{r_\alpha}$$

Four degrees of freedom

$$\theta_x: (J_1 \gamma_{12}^2 \gamma_{34}^2 s + K_1 \gamma_{12}^2 \gamma_{34}^2) \dot{\theta}_x - (K_1 \gamma_{12}^2 \gamma_{34}^2) \dot{\theta}_y = T_1 \gamma_{12} \gamma_{34}$$

$$\theta_y: -(K_1 \gamma_{12}^2 \gamma_{34}^2) \dot{\theta}_x + (D_1 \gamma_{12}^2 \gamma_{34}^2 s + K_2 \gamma_{12}^2 \gamma_{34}^2) \dot{\theta}_y - (D_1 \gamma_{12}^2 \gamma_{34}^2 s) \dot{\theta}_2 = 0$$

$$\theta_2: -(D_1 \gamma_{12}^2 \gamma_{34}^2 s) \dot{\theta}_y + (J_2 \gamma_{34}^2 s^2 + (D_1 \gamma_{12}^2 \gamma_{34}^2 + D_2 \gamma_{34}^2) s + K_2 \gamma_{34}^2) \dot{\theta}_2 - (D_2 \gamma_{34}^2 s + K_2 \gamma_{34}^2) \dot{\theta}_o = 0$$

$$\theta_o: -(D_2 \gamma_{34}^2 s + K_2 \gamma_{34}^2) \dot{\theta}_2 + (J_2 \gamma_{34}^2 + J_4) s^2 + (D_2 \gamma_{34}^2 + D_3) s + K_2 \gamma_{34}^2 + K_3) \dot{\theta}_o = 0$$

Note θ_x , θ_y , and θ_2 are just dummy variables; they are proportional to angles in the system as follows

Element
J₁, K₁ left

K₁ right, D₁ left

D₁ right

J₂, D₂ left, K₂ left

D₂ right, K₂ right, J₃

J₄, D₃ "top", K₃ left

Actual rotation

$\theta_x \gamma_{12} \gamma_{34}$

$\theta_y \gamma_{12} \gamma_{34}$

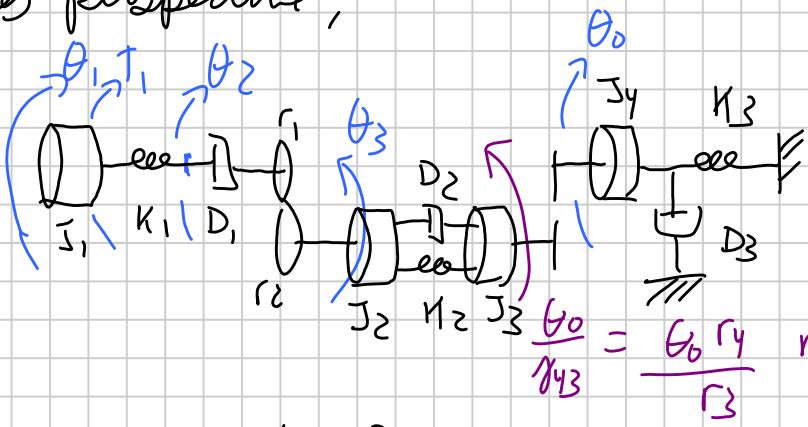
$\theta_2 \gamma_{12} \gamma_{34}$

$\theta_2 \gamma_{34}$

$\theta_o \gamma_{34}$

θ_o

Note: multiple ways to solve; could write from each angle's perspective;



$$\frac{\theta_0}{\theta_{43}} = \frac{g_0 r_y}{r_3} \text{ not independent of } \theta_0$$

$$\theta_1: (J_1 s^2 + K_1) \dot{\theta}_1 - (K_1) \ddot{\theta}_2 = T_1$$

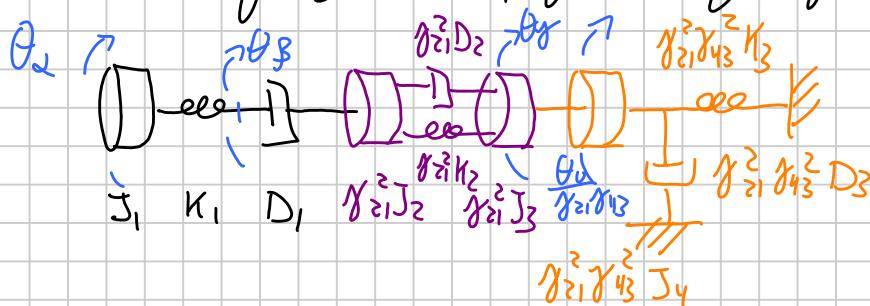
$$\theta_2: -(K_1) \dot{\theta}_1 + (D_1 s + K_1) \dot{\theta}_2 - (D_1 s) \frac{\theta_3}{J_{12}} = 0$$

$$\theta_3: -(\gamma_{12}^2 D_1 s) \frac{\theta_2}{J_{12}} + (J_2 s^2 + (\gamma_{12}^2 D_1 + D_2)s + K_2) \dot{\theta}_2 - (D_2 s + K_2) \frac{\theta_0}{J_{13}} = 0$$

$$\theta_0: -(\gamma_{34}^2 D_2 s + \gamma_{34}^2 K_2) \frac{\dot{\theta}_3}{J_{34}} + ((\gamma_{34}^2 J_3 + J_4)s^2 + (\gamma_{34}^2 D_2 + D_3)s + \dots - (\gamma_{34}^2 K_2 + K_3)) \dot{\theta}_0 = 0$$

works as well.

Could also reflect to J1 frame of reference.



IV)



$$J_m = J_a + J_1 + \gamma_{21}^2 (J_2 + J_b + J_3 + M_c r^2)$$

$$D_m = D_a + D_1 + \gamma_{21}^2 (D_2 + D_3 + F_v r^2)$$

$$\frac{\Theta_m}{E_a} = \frac{\frac{1}{J_m} \frac{K_t}{R_a}}{S \left(S + \frac{1}{J_m} (D_m + \frac{K_t K_b}{R_a}) \right)}$$

$$\frac{X}{r \gamma_{21}} = \Theta \quad \frac{X}{E_a} = \frac{r \gamma_{21} \frac{1}{J_m} \frac{K_t}{R_a}}{S \left(S + \frac{1}{J_m} (D_m + \frac{K_t K_b}{R_a}) \right)}$$