

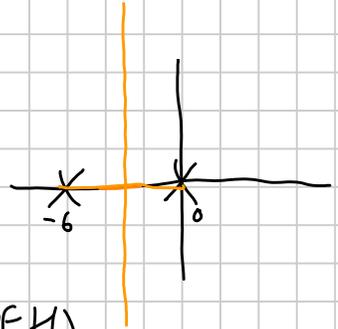
Controls F15 Test III Solutions

Note Title

I 1) $G = \frac{1}{(s)(s+6)} = \frac{1}{s^2+6s}$ $H=1$
 $T = \frac{KG}{1+KG} = \frac{K}{s^2+6s+K}$
 $G_{eq} = KG = \frac{K}{s^2+6s}$ since $H=1$

2 (a) $\begin{matrix} s^2 & 1 & K \\ s^1 & 6 & \\ s^0 & K & \end{matrix}$ $K > 0$

3 (b) OLP: 0, -6
 OLZ: none
 2 asymptotes; $\sigma_a = \frac{-0-6}{2} = -3$
 $\theta_a = \pm 90^\circ$
 never marginally stable $K > 0$



1 (c) (LZ: none (no zeros of G or poles of H))

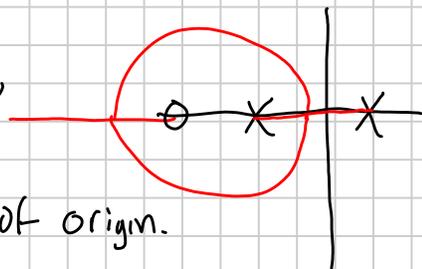
3 (d) Type 1, $K_V = \lim_{s \rightarrow 0} sG_{eq} = \frac{K}{6}$, $e_{ss} = \frac{1}{K_V} = \frac{6}{K}$

2) $G = \frac{s+4}{(s-1)(s+2)} = \frac{s+4}{s^2+s-2}$ $H=1$
 $T = \frac{KG}{1+KG+H} = \frac{K(s+4)}{(s-1)(s+2) + K(s+4) + 1} = \frac{K(s+4)}{s^2+s-2+Ks+4K} = \frac{K(s+4)}{s^2+(K+1)s+4K-2}$

$G_{eq} = KG$ since $H=1$

(a) $\begin{matrix} s^2 & 1 & 4K-2 \\ s^1 & K+1 & \\ s^0 & 4K-2 & \end{matrix}$ $K > -1$
 $K > 1/2$ $K > 1/2$

(b) OLP: 1, -2
 OLZ: -4
 one asymptote, σ_a irrelevant, $\theta_a = 180^\circ$
 when $K = 1/2$, $(\frac{3}{2})s = 0$ so $w=0$



note: means break must be left of origin.

(c) (LZ: -4 zero of G)

(d) Type 0, $K_P = 4K/(-2) = -2K$, $e_{ss} = \frac{1}{1+K_P} = \frac{1}{1-2K}$

$$(3) \quad G = \frac{1}{(s-1)(s+4)(s+9)} = \frac{1}{s^3 + 12s^2 + 23s - 36} \quad H = 1$$

$$T = \frac{KG}{1+KGH} = \frac{K}{s^3 + 12s^2 + 23s - 36 + K} \quad G_{eg} = KG \text{ since } H=1$$

(a)
$$\begin{array}{r|l} s^3 & 1 & 23 \\ s^2 & 12 & K-36 \\ s^1 & & \\ s^0 & K-36 & \end{array} \quad \begin{array}{l} \frac{(1)(K-36) - (12)(23)}{-12} \\ \frac{312-K}{12}, K < 312 \\ K > 36 \end{array}$$

(b) OLP: 1, -4, -9
 OLZ: none
 2 asymptotes, $\sigma_a = \frac{1-4-9}{2} = -4$
 $\theta_a = 60^\circ, 180^\circ, -60^\circ$
 $K=36, s=0, K=312, 12s^2 + 276 = 0, s = \pm \frac{j\sqrt{23}}{4} = \pm j4.8$

(c) CLZ: none

(d) Type 0, $K_p = \lim_{s \rightarrow 0} G_{eg} = \frac{-K}{36}, \text{ ess} = \frac{1}{1+K_p} = \frac{1}{1-\frac{K}{36}} = \frac{36}{36-K}$

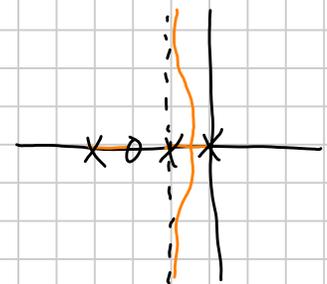
$$(4) \quad G = \frac{1}{(s)(s+3)} = \frac{1}{s^2+3s} \quad H = \frac{s+2}{s+1}$$

$$T = \frac{KG}{1+KGH} = \frac{\frac{K}{s^2+3s}}{1 + \frac{K(s+2)}{s^3+4s^2+3s}} = \frac{K(s+1)}{s^3+4s^2+3s+Ks+2K} = \frac{K(s+1)}{s^3+4s^2+(K+3)s+2K}$$

$$G_{eg} = \frac{T}{1-T} = \frac{\frac{K(s+1)}{s^3+4s^2+(K+3)s+2K}}{1 - \frac{K(s+1)}{s^3+4s^2+(K+3)s+2K}} = \frac{K(s+1)}{s^3+4s^2+3s+K}$$

(a)
$$\begin{array}{r|l} s^3 & 1 & K+3 \\ s^2 & 4 & 2K \\ s^1 & & \\ s^0 & 2K & \end{array} \quad \begin{array}{l} \frac{2K-4(K+3)}{-4} = \frac{-2K-12}{-4} = \frac{K+6}{2} \\ K > -6 \end{array}$$

(b) OLP: 0, -1, -3
 OLZ: -2
 2 asymptotes, $\sigma_a = \frac{0-1-3+2}{2} = -1$
 $\theta_a = \pm 90^\circ$
 never marginal for $K > 0$



(c) CLZ: -1 (pole of H)

(d) Type 0 if $K \neq 0$ and $K=0$ is trivial
 So Type 0!

$$K_p = \lim_{s \rightarrow 0} G_{eg} = 1 \quad \text{ess} = \frac{1}{1+K_p} = \frac{1}{1+1} = 0.5$$

$$\text{II) } G_p = \frac{1}{(s-1)(s+4)(s+9)} = \frac{1}{s^3 + 12s^2 + 23s - 36}$$

$$T = \frac{KG_p}{1+KG_p} = \frac{K}{s^3 + 12s^2 + 23s - 36 + K}$$

(1) yes, $\sigma = -1.1$ is $\beta = 1$

$$K = \frac{-1}{GH} = 48.1$$

(2) When ~~just~~ underdamped, $\sigma = -1.1$ and $T_s \approx 4/|\sigma| = 3.60$

(3) $\%OS = 10$, $\beta = \frac{-\ln(.1)}{\sqrt{\pi^2 + \ln^2(.1)}} = 0.5911$

$$\zeta \omega_n \approx \frac{4}{T_s} = 2.6 \quad \omega_n = \frac{2.6}{0.5911} = 4.51$$

$$\omega_d = \omega_n \sqrt{1 - \zeta^2} = 3.64$$

$$s = -\zeta \omega_n \pm j \omega_d = -2.6 \pm j 3.64$$

(4) $G_c G_p = \frac{K(s+z_c)}{(s-1)(s+4)(s+9)}$

$$\angle G_c G_p \Big|_{s=-2.66 + j3.64} = 0^\circ + \angle s+z_c - \angle s-1 - \angle s+4 - \angle s+9 = 180$$

$$0^\circ + \angle s+z_c - 135.2^\circ - 69.8^\circ - 29.9^\circ = 180 + n360$$

$$\angle s+z_c = 234.9 - 180 = 54.9$$

$$\frac{3.64}{z_c - 2.66} = \tan(54.9) = 1.422 \quad z_c = \frac{3.64}{1.422} + 2.66 = 5.22$$

$$K = \frac{-1}{GH} \Big|_{s=-2.66 + j3.64} = \frac{-(s-1)(s+4)(s+9)}{(s+5.22)} \Big|_{s=-2.66 + j3.64} = 32.9$$

So $G_c = 32.9 (s+5.22)$ PD control

(5)

With a passing controller, better chance of
2nd order approx if lead 0 is on OLP to right
of PD zero; more to 4 (can't move to 9...)

$$\text{then, } \angle \frac{(s+4)}{(s-1)(s+4)(s+9)(s+p)} = 180 + 360n$$

$$\text{so } 0 - \angle s-1 - \angle s+9 - \angle s+p = 180 + 360n$$

$$-135^\circ - 29.9^\circ - \angle s+p = 180 + 360n$$

$$\angle s+p = 14.9^\circ$$

$$\frac{364}{p-2.66} = \tan 14.9^\circ = 0.266$$

$$p = \frac{364}{0.266} + 2.66 = 16.34$$

$$K = \frac{-1}{GH} = \frac{-(s-1)(s+9)(s+16.34)}{1} = 534$$

$$\text{so } G_c = 534 \frac{(s+4)}{(s+12.6)}$$

Lead Control.

IV)

$$G = \frac{1}{(s)(s+6)} = \frac{1}{s^2+6s} \quad H=1$$

$$1 = \frac{KG}{1+KG} = \frac{K}{s^2+6s} = \frac{K}{s^2+6s+K}$$

$$G_{eq} = KG = \frac{K}{s^2+6s} \quad \text{since } H=1$$

(1) yes, when $\sigma = -3$ so $K = \frac{-1}{GH} = \frac{-1}{\left(\frac{1}{(-3)(3)}\right)} = 9$

(2) $T_s \approx \frac{4}{3}$ since this is as far left as dom. poles go

(3) $\zeta \omega_n = 3$, 20% OS means $\zeta = \frac{-\ln(0.2)}{\sqrt{\pi^2 + \ln(0.2)^2}} = 0.456$

$$\omega_n = 3/\zeta = 6.58 \quad \omega_d = \omega_n \sqrt{1-\zeta^2} = 5.86$$

$$s = -3 \pm j 5.86$$

$$K = \frac{-1}{GH|_s} = 43.3$$

(4) $K \frac{(s+z_c)}{(s+p_c)}$ where $z_c \ll 3$ and $\frac{z_c}{p_c} = 3$

many solutions ... $K \approx$ the same

lag control

(5) Type 1, $K_p = \lim_{s \rightarrow 0} G_c G_p = \frac{(43.3)(3p_c)}{(6)(p_c)} = 21.65$

$$e_{ss} = \frac{1}{K_p} = 0.047, \quad r(t) = t u(t)$$

(6) $\frac{K(s+z_c)}{(s)}$ $z_c \ll 3$

PI control

$K \approx$ the same if z_c small enough

(7) Type 2, $K_a = \lim_{s \rightarrow 0} s^2 G_c G_p = \frac{(43.3)(z_c)}{(6)} = 7.21 z_c$

$$e_{ss} = \frac{1}{K_a} = \frac{0.139}{z_c}, \quad r(t) = \frac{1}{2} t^2 u(t)$$