Instructions

First - please turn off any cell phones or other annoyance-producing devices. Vibrate mode is not enough - your device needs to be in a mode where it will make no sounds during the course of the test, including the vibrate buzz or those acknowledging receipt of a text or voicemail.

Please be sure to put each problem on its own page or pages - do not write answers to more than one problem on any piece of paper and do not use the back of a problem for work on a different problem. You will be turning in each of the problems independently. This cover page should be stapled to the front of Problem 1.

Make sure that your name and NET ID are clearly written at the top of every page, just in case problem parts come loose in the shuffle. Make sure that the work you are submitting for an answer is clearly marked as such. Finally, when turning in the test, individually staple all the work for each problem and place each problem’s work in the appropriate folder.

Note that there may be people taking the test after you, so you are not allowed to talk about the test - even to people outside of this class - until I send along the OK. This includes talking about the specific problem types, how long it took you, how hard you thought it was - really anything. Please maintain the integrity of this test.

You may use the $\parallel$ symbol for impedances in parallel and do not need to expand that construction (unless you are required to get a numerical answer). Be clear with your use of parentheses, however; simply writing something like

\[ R_{eq} = R_1 + R_2 \parallel R_3 + R_4 \]

is too vague since it could refer to any of the four combinations below:
Problem I: [25 pts.] Switched Circuits I

Given the following circuit:

\[ R_2 \quad v_a \quad R_1 \quad v_C \quad C \quad i_b \quad R_3 \]

Note that the left switch opens at time \( t=0 \) s and, four seconds later, the right switch closes.

(a) Determine the values below symbolically:
- \( i_C(0^+) \)
- \( v_C(0^+) \)
- \( i_C(4^+) \)
- \( v_C(4^+) \)
- \( i_C(\infty) \)
- \( v_C(\infty) \)

(b) Assuming \( v_a = 10 \) V, \( i_b = 250 \) \( \mu \)A, \( L = 8 \) mH, \( C = 100 \) \( \mu \)F, \( R_1 = 1 \) k\( \Omega \), \( R_2 = 2 \) k\( \Omega \), and \( R_3 = 20 \) k\( \Omega \), give an expression for \( v_C(t) \) for \( t > 0 \) s. Accurately sketch the voltage across the capacitor for the first 10 seconds after the left switch is opened.
Problem II: [25 pts.] Filter Design

(1) Design a voltage-to-voltage bandpass filter with a maximum gain of 1 at the resonant frequency that has half-power frequencies at $\omega=250$ rad/s and $\omega=25000$ rad/s. Unfortunately, the only capacitors you can find in the lab are 4.7 $\mu$F, but you have adjustable resistors and inductors at your disposal. As part of the design, specify the value of the resistor and the inductor as well as the filter’s:

(a) Bandwidth
(b) Damping ratio
(c) Linear center frequency
(d) Natural frequency
(e) Quality factor

(2) Make an accurate sketch of the Bode magnitude plot for the filter. Be sure to clearly label the axes. Your plot must extend at least two decades beyond the half-power frequencies in either direction.
Problem III: [25 pts.] Transfer Functions

Given the following circuit:

and assuming the values for the passive elements are known,

(a) Find an expression for the following transfer function:

\[ G(j\omega) = \frac{V_x(j\omega)}{V_a(j\omega)} \]

(b) What kind of filter do you think \( G(j\omega) \) represents? Why do you think that?

(c) Assuming

\[
\begin{align*}
R_1 &= 100 \ \text{k}\Omega \\
C_1 &= 100 \ \mu\text{F} \\
R_2 &= 20 \ \text{k}\Omega \\
C_2 &= 100 \ \mu\text{F}
\end{align*}
\]

and

\[ v_a(t) = 3 + 2 \cos(2t + 15^\circ) - 4 \sin(10t - 35^\circ) \ \text{V} \]

find an expression for the steady-state voltage across the resistor, \( v_x(t) \).
Problem IV: [25 pts.] Switched Circuits II

Given the following circuit:

\[ R_1 \quad L \quad R_2 \quad v_C \quad C \quad R_3 \quad i_b \]

and assuming that the values for the passive elements as well as sources \( v_a \) and \( i_b \) are known

(1) Assuming \( v_a \) and \( i_b \) are constant values for a long time before \( t = 0 \) s, determine

- \( i_L(0^-) \)
- \( v_L(0^-) \)
- \( i_C(0^-) \)
- \( v_C(0^-) \)
- \( i_L(0^+) \)
- \( v_L(0^+) \)
- \( i_C(0^+) \)
- \( v_C(0^+) \)

(2) Determine (but do not solve) a differential equation for the inductor current \( i_L(t) \) in terms of the passive element values and the sources \( v_a(t) \) and \( i_b(t) \) for \( t > 0 \) s. Note that \( v_C(t) \) cannot be a part of the final answer.