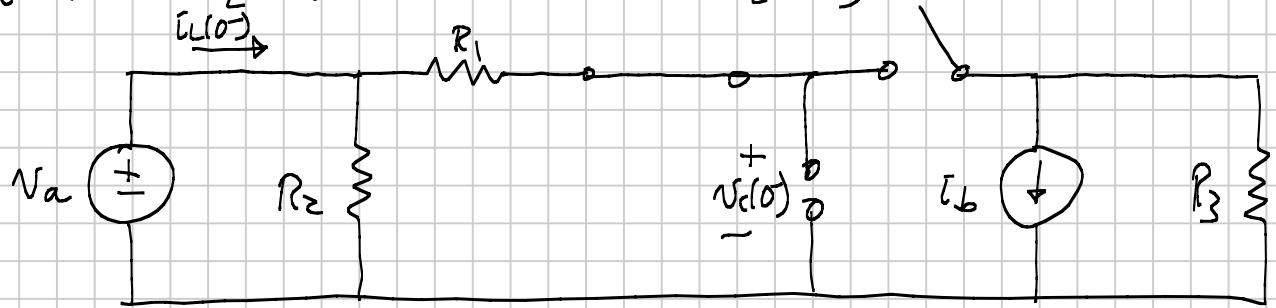


BME153 Test 2 Spring 2009

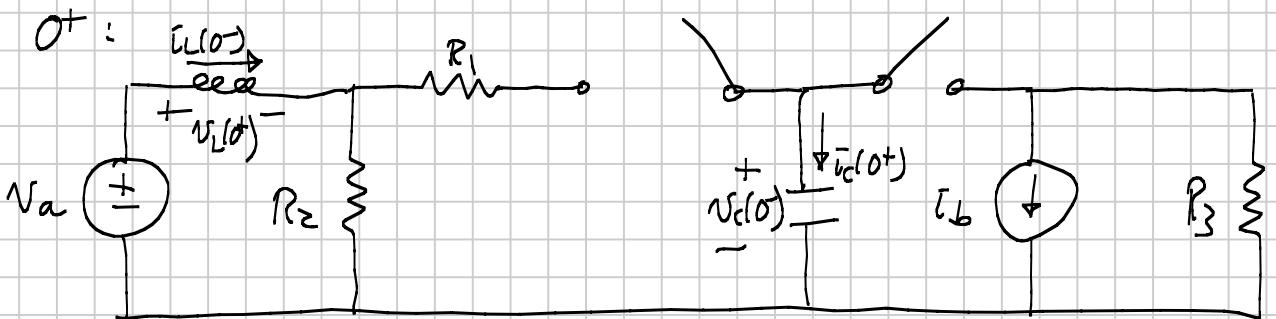
Note Title

I) $v_c(0^+)$ and $\bar{i}_L(0^+) = v_c(0^-)$ and $\bar{i}_L(0^-)$:



$$v_c(0^-) = \underline{V_a} \quad \bar{i}_c(0^-) = 0$$

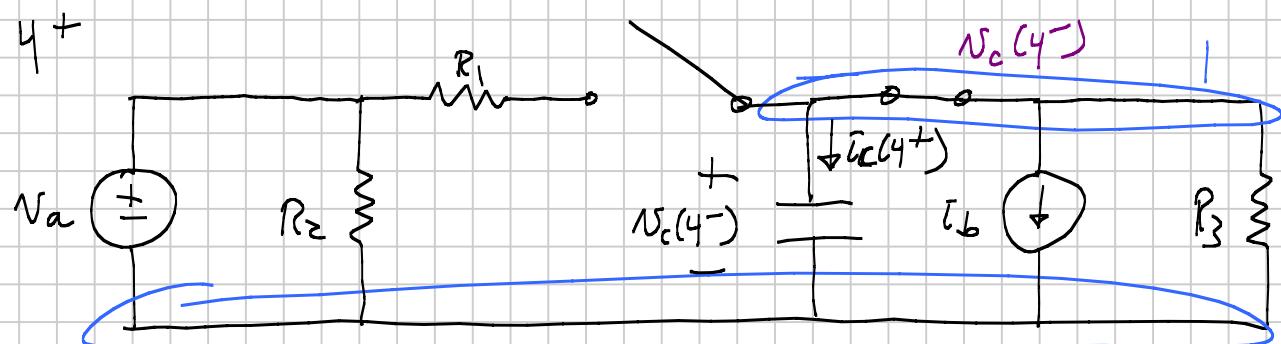
$$\bar{i}_L(0^-) = \frac{V_a}{R_2}$$



$$v_c(0^+) = v_c(0^-) = \underline{V_a}$$

$$\bar{i}_c(0^+) = 0$$

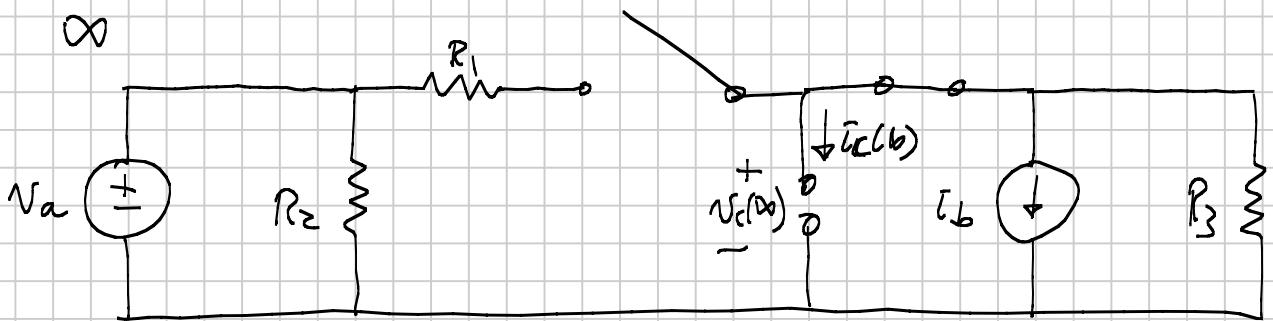
Same at 4-
since no current through \bar{i}_c



$$v_c(4^+) = v_c(4^-) = \underline{V_a}$$

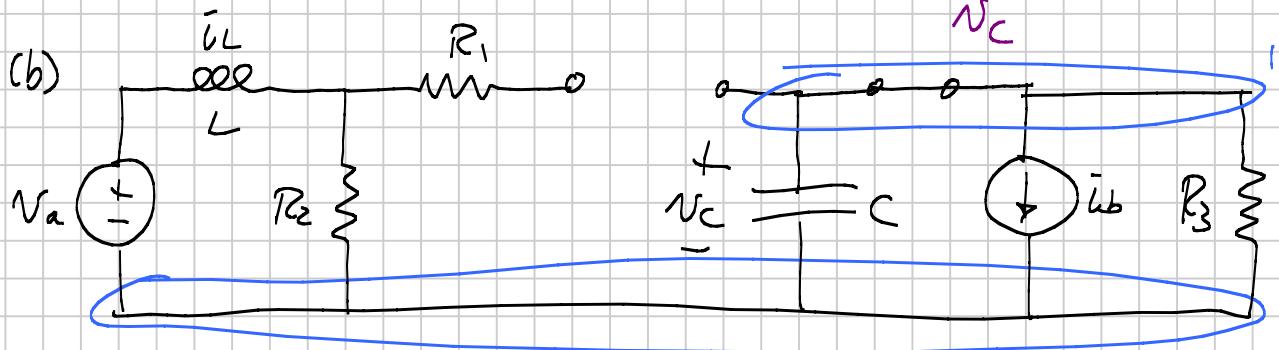
$$KCL, 1: \bar{i}_c(4^+) + \bar{i}_b + \frac{v_c(4^-)}{R_3} = 0$$

$$\bar{i}_c(4^+) = -\bar{i}_b - \frac{v_c(4^-)}{R_3} = -\bar{i}_b - \frac{\underline{V_a}}{R_3}$$



$$V_c(\infty) = -I_b R_3$$

$$I_c(\infty) = 0$$



$$C \frac{dV_c}{dt} + I_b + \frac{V_c}{R_3} = 0$$

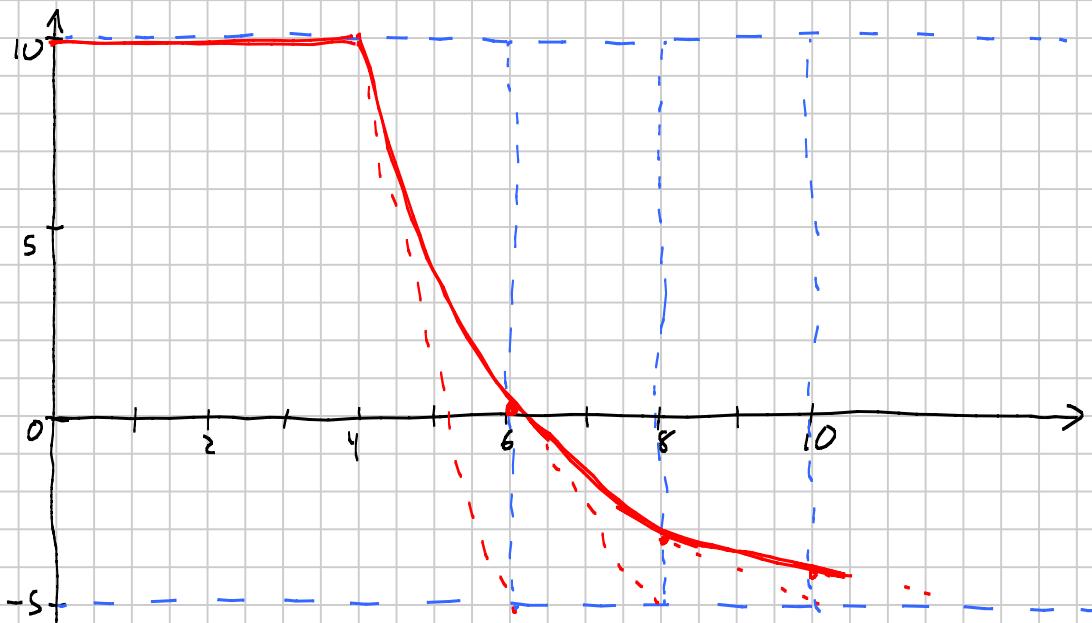
$$R_3 C \frac{dV_c}{dt} + V_c = -R_3 I_b$$

$$2 \frac{dV_c}{dt} + V_c = -5$$

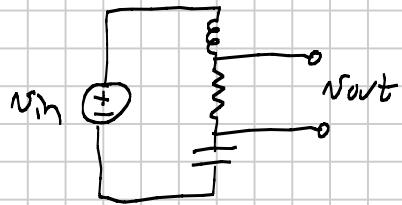
$$\begin{aligned} V_{c,i} &= V_a = 10 \\ V_{c,f} &= -5 \\ C &= 2 \end{aligned}$$

$$0 < t < 4 \quad V_c(t) = 10$$

$$t > 4 \quad V_c(t) = -5 + 15 e^{-\frac{(t-4)}{2}}$$



II) V-V BP:



$$H = \frac{V_{out}}{V_m} = \frac{R}{j\omega L + R + j\omega C} = \frac{j\omega CR}{(j\omega)^2 LC + j\omega CR + 1}$$

$$|H| = \frac{j\omega (R/L)}{(j\omega)^2 + j\omega(R/L) + 1/LC} = \frac{2\zeta w_n j\omega}{(j\omega)^2 + 2\zeta w_n j\omega + w_n^2}$$

Since HP are at 250 rad/s and 25000 rad/s,

BAND WIDTH = 24750 rad/s

$$\underline{w_n} = \text{log. avg.} = \sqrt{250 \times 25000} = 2500 \text{ rad/s}$$

$$\underline{\frac{50}{s}} \quad \frac{1}{LC} = 2500^2 \quad L = \frac{1}{2500^2 \cdot C} = .034043 \text{ H}$$

$$\underline{BW} = \frac{R}{L} = 24750 \quad \underline{\text{so}} \quad R = 24750 L = 842.6 \text{ } \underline{\text{JZ}}$$

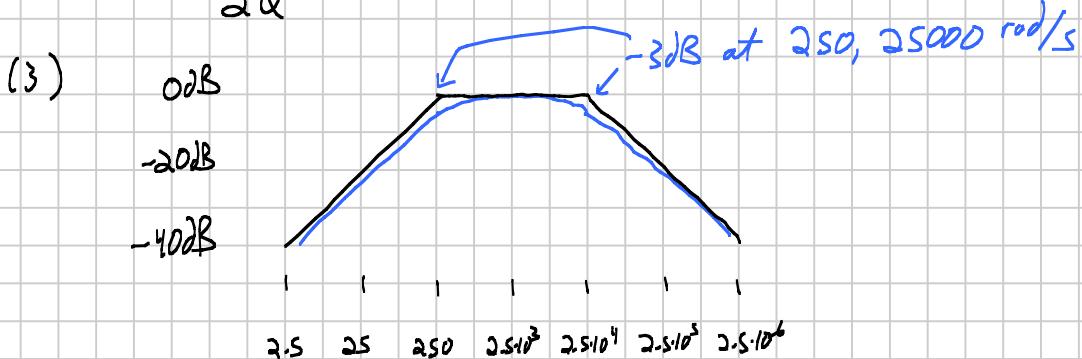
(2) $\underline{BW} = 24750 \text{ rad/s as above}$

$$Q = \frac{w_n}{BW} = \frac{2500}{24750} = .10101$$

$w_n = 2500 \text{ rad/s as above}$

$$w_{lman} = \frac{1}{2} (250 + 25000) = 12625 \text{ rad/s}$$

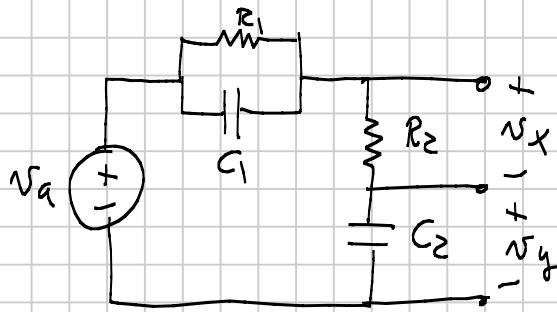
$$\underline{b} = \frac{1}{2Q} = 4.95$$



Note: $\frac{j\omega 25250}{(j\omega + 250)(j\omega + 25000)}$ is an acceptable approximation with

$$BW = 25250, w_n = 2500, Q = .099, b = 5.05, w_c = 12871, w_{co} = 246, 25496$$

III)



$$Z_{Top} = R_1 \parallel \frac{1}{j\omega C_1} = \frac{R_1}{1 + j\omega C_1 R_1}$$

$$G = \frac{V_x}{V_a} = \frac{R_2}{Z_{Top} + R_2 + \frac{1}{j\omega C_2}} = \frac{R_2(j\omega C_2)(1 + j\omega C_1 R_1)}{R_1(j\omega C_2) + R_2(j\omega C_2)(1 + j\omega C_1 R_1) + 1(1 + j\omega C_1 R_1)}$$

$$G = \frac{(j\omega)^2 C_1 C_2 R_1 R_2 + (j\omega) C_2 R_2}{(j\omega)^2 C_1 C_2 R_1 R_2 + j\omega(C_2 R_1 + C_2 R_2 + C_1 R_1) + 1}$$

(b) High pass; $\lim_{w \rightarrow 0} |G| = 0$ and $\lim_{w \rightarrow \infty} |G| = 1$

$$(c) G = \frac{(j\omega)^2 20 + (j\omega) 2}{(j\omega)^2 20 + (j\omega) 22 + 1}$$

$$G(j0) = 0 \quad G(j2) = \frac{-80 + j4}{-80 + j44 + 1} = .886 \angle -45.8 \text{ rad}$$

$$G(j10) = \frac{-2000 + j20}{-2000 + j220 + 1} = .995 \angle -0.996 \text{ rad}$$

0 rad/s

$$3 \cdot 0 = 0$$

2 rad/s

$$(2 \angle 15^\circ)(-886 \angle -45.8 \text{ rad}) = 1.772 \angle -72 \text{ rad} \\ = -1.772 \angle 41.25^\circ$$

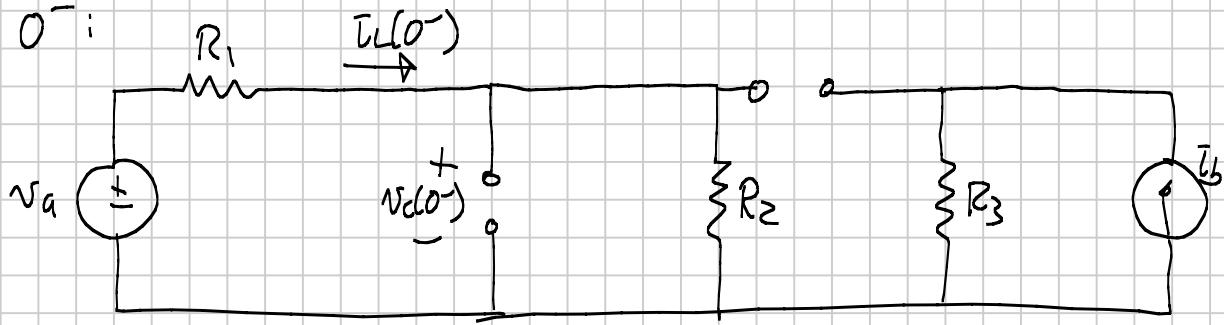
10 rad/s

$$-4 \sin(10t - 35^\circ) = 4 \sin(10t + 145^\circ) = 4 \cos(10t + 55^\circ)$$

$$(4 \angle 55^\circ)(.995 \angle .0996 \text{ rad}) = 3.978 \angle 1.06 \text{ rad} \\ = 3.978 \angle 60.73^\circ$$

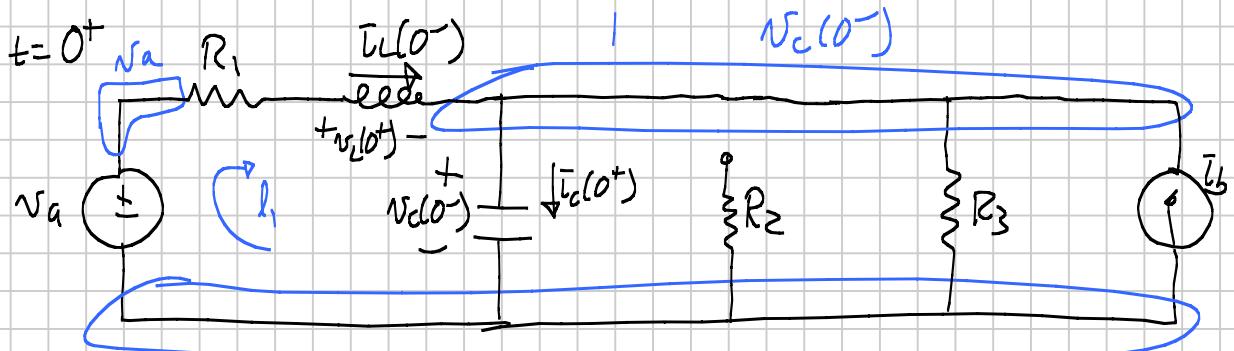
$$v_u(t) = 0 + 1.772 \cos(2t + 41.25^\circ) + 3.978 \cos(10t + 60.73^\circ) \text{ V}$$

IV) 0^- :



$$\bar{I}_L(0^-) = \frac{V_a}{R_1 + R_2} \quad V_c(0^-) = \frac{R_2}{R_1 + R_2} V_a$$

$$V_L(0^-) = 0 \quad I_c(0^-) = 0$$



$$V_c(0^+) = V_c(0^-) = \frac{R_2}{R_1 + R_2} V_a$$

$$\bar{I}_L(0^+) = \bar{I}_L(0^-) = \frac{V_a}{R_1 + R_2}$$

$$KCL_{A_1}: -\bar{I}_L(0^-) + \bar{I}_c(0^+) + \frac{V_c(0^-)}{R_3} - \bar{I}_b = 0$$

$$\bar{I}_c(0^+) = \bar{I}_L(0^-) - \frac{V_c(0^-)}{R_3} + \bar{I}_b$$

$$= \frac{V_a}{R_1 + R_2} - \frac{R_2 V_a}{(R_1 + R_2) R_3} + \bar{I}_b$$

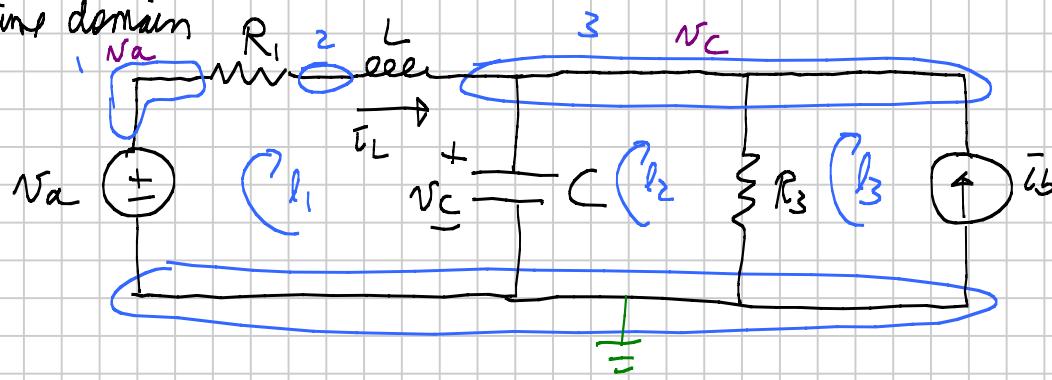
$$= \frac{(R_3 - R_2) V_a + R_3 (R_1 + R_2) \bar{I}_b}{(R_1 + R_2) R_3}$$

$$KVLL: -V_a + R_1 \bar{I}_L(0^-) + V_L(0^+) + V_c(0^-) = 0$$

$$V_L(0^+) = V_a - R_1 \bar{I}_L(0^-) - V_c(0^-)$$

$$= V_a - \frac{R_1 V_a}{R_1 + R_2} - \frac{R_2 V_a}{R_1 + R_2} = 0$$

$t > 0$ Time domain



$$KCL_{n_3}: -\bar{I}_L + C \frac{dV_C}{dt} + \frac{V_C}{R_3} - \bar{I}_b = 0$$

$$KVL_{l_1}: -V_a + R_1 \bar{I}_L + L \frac{d\bar{I}_L}{dt} + V_C = 0$$

$$V_C = V_a - L \frac{d\bar{I}_L}{dt} - R_1 \bar{I}_L$$

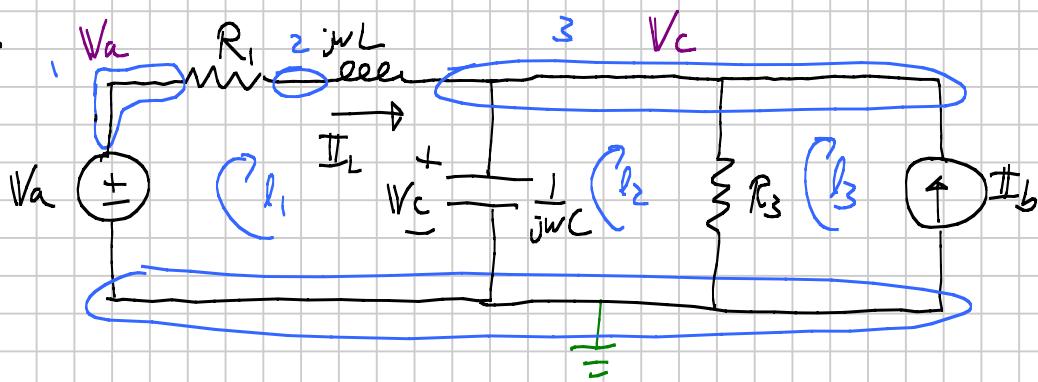
$$-\bar{I}_L + C \frac{d}{dt} \left(V_a - L \frac{d\bar{I}_L}{dt} - R_1 \bar{I}_L \right) + \frac{1}{R_3} \left(V_a - L \frac{d\bar{I}_L}{dt} - R_1 \bar{I}_L \right) - \bar{I}_b = 0$$

$$-\bar{I}_L + C \frac{dV_a}{dt} - LC \frac{d^2\bar{I}_L}{dt^2} - R_1 C \frac{d\bar{I}_L}{dt} + \frac{V_a}{R_3} - \frac{L}{R_3} \frac{d\bar{I}_L}{dt} - \frac{R_1}{R_3} \bar{I}_L - \bar{I}_b = 0$$

$$LC \frac{d^2\bar{I}_L}{dt^2} + \left(R_1 C + \frac{L}{R_3} \right) \frac{d\bar{I}_L}{dt} + \left(1 + \frac{R_1}{R_3} \right) \bar{I}_L = \left(\frac{dV_a}{dt} + \frac{V_a}{R_3} - \bar{I}_b \right)$$

or

freq.



$$KVL_{l_1}: -V_a + R_1 I_L + jwL I_L + V_C = 0$$

$$V_C = V_a - R_1 I_L - jwL I_L$$

$$KCL_{n_3}: -I_L + jwC V_C + \frac{V_C}{R_3} - I_b = 0$$

$$I_L - \left(jwC + \frac{1}{R_3} \right) \left(V_a - R_1 I_L - jwL I_L \right) = -I_b$$

$$\left((jw)^2 LC + jw \left(CR_1 + \frac{L}{R_3} \right) + 1 + \frac{R_1}{R_3} \right) I_L = jwC V_a + \frac{V_a}{R_3} - I_b$$

$$LC \frac{d^2 I_L}{dt^2} + \left(CR_1 + \frac{L}{R_3} \right) \frac{dI_L}{dt} + \left(1 + \frac{R_1}{R_3} \right) I_L = \left(\frac{dV_a}{dt} + \frac{V_a}{R_3} - I_b \right)$$